SYSTEM DESIGN AND CONTROL INTEGRATION FOR ADVANCED MANUFACTURING

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HAN-XIONG LI
XINJIANG LU

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PREFACE

The manufacturing industry has changed drastically from the traditional industry, like steel and auto factory, to the semiconductor or IC industry of the 1980s, and to the internet-based global manufacturing nowadays. Advanced manufacturing involves the use of technology to improve products and/or processes, with the relevant technology being described as “advanced,” “innovative,” or “cutting edge.” The distinctions between traditional manufacturing and advanced manufacturing are in terms of volume and scale economies, labor and skill content, and the depth and diversity of the network surrounding the industry, and intelligence added in the system. No matter how complex or advanced the manufacturing operation is, it always consists of basic actions offered by basic systems that can be categorized as static systems, dynamic systems, and their combination. Performance of each basic system will be crucial to the overall performance of the manufacturing.

In order to design and manufacture these high quality products at lower costs, accurate mechanical systems are crucial in the manufacturing industry. One problem is inconsistent performance caused by uncontrollable variations in manufacturing operations, material properties, and complex operating environments. This inconsistent performance often results in a failure in operation. Thus, robust performance, insensitive to all possible changes in demand, model uncertainties, and external disturbance, is one of the most important concerns in the design and control of these systems.

Robust design and its integration with control are the most important methods commonly used to achieve robust performance of the system. The studies of robust design and its integration with control are becoming increasingly important. In the last few decades, there have been many studies on robust design and its integration with control. There are still many unsolved problems. The purpose of this book is to provide a brief view of the previous work on robust design and its integration with control, and develop new design and integration methods to tackle some of these unsolved problems.

In this book, a systematic overview and classification is first presented on robust analysis/design for static and dynamic systems, and the integration of design and control. Limitations and advantages of various approaches are also discussed. Next, three novel robust design approaches are proposed for design of the static system: the variable sensitivity based robust design approach for small-scale parameter variation, the multi-domain modeling-based robust design for large-scale parameter variation, and the hybrid model/data-based robust design for both parameter variation and model uncertainty. Then, the robust eigenvalue design methods are developed to maintain both stability and robustness of the dynamic system under parameter variation and model uncertainty. Finally, two novel methods are proposed for integrating design and control for the hybrid system under parameter uncertainty. One method is for the dynamic system with hybrid discrete/continuous variables. An easily controlled dynamic behavior will first be obtained through the process design, and then integrated with control under the robust pole assignment. Another method is for the hybrid system working in a large region with an unmeasured overall performance. A low level process control will be integrated with high level system design with the help of fuzzy modeling method, and optimized with the particle swarm optimization (PSO) method. All the methods presented in this book have been successfully applied to the design of some mechanical equipment and the curing process in IC packaging, and are applicable to a wide range of systems in the manufacturing industry.

The book will be of great benefit to undergraduate and postgraduate students in many disciplines, including manufacturing engineering, mechanical engineering, electrical engineering, and control engineering. The book is also intended for researchers, research students and application engineers interested in robust design and its integration with control.

HAN-XIONG LI

City University of Hong Kong
Central South University

XINJIANG LU

Central South University

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PART I

BACKGROUND AND FUNDAMENTALS

CHAPTER 1

INTRODUCTION

This chapter is an introduction of the book. It briefly introduces background, motivation, and objectives of the research, followed by the contribution and organization of the book.

1.1 BACKGROUND AND MOTIVATION

Since we moved into the Industrial Age, most of the products used have been manufactured by machines and production lines. The manufacturing industry has changed much from the traditional sector, like steel and auto factory, to the semiconductor or IC industry in 1980s, and to the internet-based global manufacturing nowadays. Advanced manufacturing uses the so-called “advanced,” “innovative,” or “cutting-edge” technology to improve products and/or processes. The distinctions between traditional sectors of manufacturing and advanced manufacturing are in terms of volume and scale economies, labor and skill content, and intelligence added in the system.

In modern IC industry, the higher speed, the higher precision, and the higher intelligence have become common requirements to many of the processes involved, for example, epoxy/silicone dispensing (Li et al., 2007), curing process (Li, Deng, and Zhong, 2004; Deng, Li, and Chen, 2005), bonding/wiring process (Li and Zuo, 1999), and so on. Even in a traditional industry, like the forging press machine (Lu, Li, and Chen, 2012), the machine will seek help from an intelligence unit for meeting quality

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and economic constraints. Modern information technology can make a traditional system more advanced.

No matter how complex or advanced the manufacturing operation is, it always consists of basic actions offered by basic systems. These basic systems could be classified into the following three different categories.

- ***The static system.*** The performance is invariant over time, so it is discrete.
- ***The dynamic system.*** The performance is varying over time, so it is continuous.
- ***The hybrid system.*** It is a combination of the above two, which forms a hybrid system with discrete/continuous parameters, or a hybrid discrete/continuous system.

Design for advanced manufacturing is actually centered on the design and control of these basic systems, as the performance of every basic system is crucial to the overall performance of the manufacturing.

Since advanced manufacturing usually involves more complex system configuration and more advanced technologies, it will require a higher quality design of each basic system involved in the operation. However, unavoidable external variations in manufacturing operations, material properties, and a complex operating environment will result in an inconsistent performance of the system, which will be a big challenge to design for manufacturing. If these variations are not properly considered in product design, the degraded performance may result in a failure in operation (Caro, Bennis, and Wenger, 2005). Thus, robust performance, insensitive to all possible changes in demand, model uncertainties, and external disturbance, is one of the most important concerns in the design of any system.

In system design, robust design is the most important method commonly used to achieve robust performance. Its fundamental principle is minimizing the sensitivity of the performance to uncontrollable variations. Most of these approaches are for static systems, a few for dynamic systems. Furthermore, design and control are always separated in both academic research and industrial applications, which leads to few effective methods for the hybrid system.

The principal goal in this book is to develop effective design methods for fundamental systems existing in advanced manufacturing, including

1. novel robust design methods for both static and dynamic systems; and
2. robust design and control integration methods for the hybrid discrete/continuous system.

Though these methods are studied for basic systems in this book, they should be easily applied to any advanced manufacturing or production.

There are three different sets of variables that will appear in robust design.

- ***Design variable (or control variable).*** This is the controllable variable with its nominal value to be designed ideally between the upper and lower bounds. The variations around its nominal value are usually caused by poor manufacturing.

- ***Uncertainty.*** This usually includes parameter variation, noise, and model uncertainty. It cannot be adjusted by the designer, and thus is uncontrollable.
- ***Performance.*** This is the objective of the design and depends on the system model, design variable, and uncertainty.

Based on the above definition, we will introduce and discuss robust design and control integration in the rest of the chapter.

1.1.1 Robust Design for Static Systems

Robust design for the static system minimizes the influence of uncertainty on steady-state performance. Two typical robust design examples of the static system are introduced in Examples 1.1 and 1.2.

Example 1.1: Nonlinear system The damper structure widely exists in manufacturing industry and can be simplified as in Figure 1.1 (Caro, Bennis, and Wenger, 2005), where M and C_d are mass of the moving part and damping coefficient in the chamber, respectively. The excitation force $F(t)$ is assumed to be $F\cos(\omega{\cdot}t)$. The displacement will be $X(t) = X\cos(\omega \cdot t + \phi)$, where ϕ is the phase.

The performances X and ϕ can be expressed as follows:

$$X = \frac{F}{\omega\sqrt{C_d^2 + \omega^2 M^2}}, \quad \phi = \tan^{-1}\left(\frac{\omega M}{C_d}\right) \tag{1.1}$$

The objective is to keep the displacement and the phase at desirable values under the given excitation force. Due to manufacturing error, variations coming from fluid properties and the operating environment, there are large uncontrollable variations from the design variable M as well as the model parameter C_d in this system. Thus, this nonlinear system should be designed to be robust to these uncontrollable variations. ■

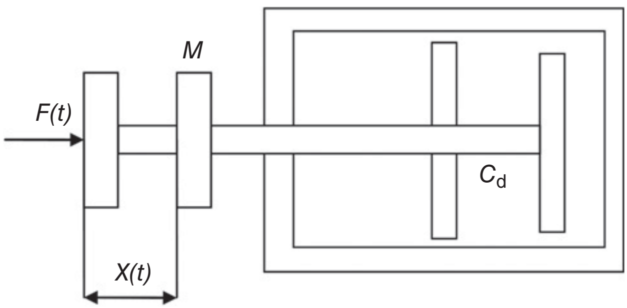

FIGURE 1.1 Damper

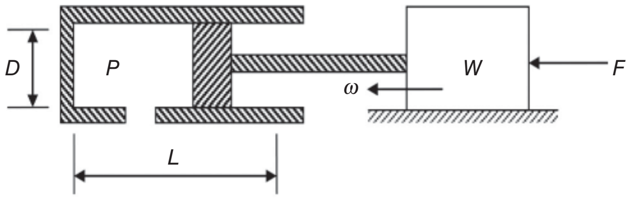

FIGURE 1.2 A pneumatic cylinder

Example 1.2: Partially unknown system The pneumatic cylinder, widely existing in manufacturing industry, is used to move a load of weight W along a horizontal surface, as shown in Figure 1.2. There exists the friction force F between the load and the surface, and the unknown disturbance force w is caused by other uncertain factors, such as leakage. The load is accelerated within a distance L to attain a steady-state velocity V. If the supply pressure is P, the actuator size D will be designed for a robust performance.

The performance V may be expressed as the sum of the known nominal model f and the model uncertainty Δf

$$V = f + \Delta f \tag{1.2}$$

with $f = \sqrt{\frac{gL(\pi D^2 P - 4F)}{2W}}$, $\Delta f = \sqrt{\frac{gL}{2W}}\left(\sqrt{\pi D^2 P - 4(F + w)} - \sqrt{\pi D^2 P - 4F}\right)$.

The nominal model f is derived from the force balance in the absence of the disturbance force w. The model uncertainty Δf is caused by unknown disturbance force w, and thus it, including its structure, is unknown as a black box to designers. For a desirable performance, a robust design is needed to properly handle all these uncertainties coming from the design variable D and the parameters W and F in the system. ■

In past decades, much effort has been dedicated to robust design of the static system. Design on this aspect can be classified into two main categories: the experiment-based robust design and the model-based robust design.

The experiment-based methods, as indicated in Figure 1.3a, design system robustness using experimental data. These methods have the advantage that no accurate system model is required. Typical examples include the Taguchi method (Ross, 1988; Taguchi, 1987, 1993) and the response surface method (Box, 1988; Tsui, 1992; Engel and Huele, 1996; Choi, 2005). All these methods are developed generally based on experiment data without process knowledge. Thus, the cost could be high if a large number of experiments are needed, and the method may not be accurate, especially for the strongly nonlinear system. Moreover, they cannot handle variations of design variables (Chen et al., 1996a). All these disadvantages may limit their applications and make it difficult to be applied for the nonlinear system described in Example 1.1, or the partially known system with variations of design variables described in Example 1.2.

The second class of methods is the model-based robust design, as shown in Figure 1.3b, which uses the model information to design the system robustness.

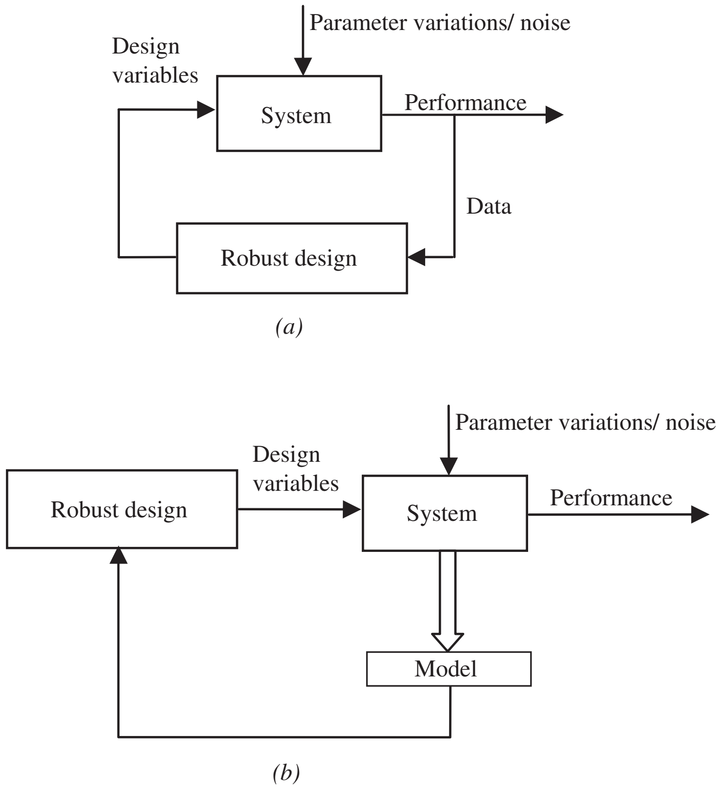

FIGURE 1.3 Traditional static robust designs: (a) experiment-based robust designs; (b) model-based robust designs

These kinds of methods are low cost and have high design accuracy compared with the experiment-based methods. In past decades, much effort has been dedicated to this class of robust design, which can be divided into two categories (Li, Azarm, and Boyars, 2006): probabilistic robust design approaches and deterministic robust design approaches.

The probabilistic robust design approaches use probabilistic information of variables, usually their mean and variance, to minimize the sensitivities of the performance (Li, Azarm, and Boyars, 2006). There are many authors that have contributed to the probabilistic approaches (e.g., Chen et al., 1996a, 2000; Al-Widyan and Angeles, 2005; Kalsi, Hacker, and Lewis, 2001; Yu and Ishii, 1998). The main shortcoming of the probabilistic approaches is that probability distributions must be known or presumed. Accurate knowledge of the distributions may be difficult to obtain and the presumed distributions may not be correct (Li, Azarm, and Boyars, 2006).

The deterministic robust design methods use a worst-case scenario approach to minimize the sensitivity. The robust design solutions are obtained using gradient information of variables, usually the Euclidean norm method and the condition number method (e.g., Ting and Long, 1996; Zhu and Ting, 2001; Caro, Bennis, and Wenger, 2005; Beyer and Sendhoff, 2007), to improve the system sensitivity. However, this gradient information may not be easy to obtain in practice.

In general, the model-based robust designs have the following limitations.

- They are mainly based on the approximate first- or second-order model derived by local linearization for the sake of simplicity and easy design. When the system is strongly nonlinear with larger uncontrollable variations, similar to Example 1.1, larger approximation errors may arise and make the design less effective.
- Ideally, the model-based design only works when the system model is available. Practically, an accurate model is often difficult to obtain due to complex boundary conditions, complexity of the process, or incomplete knowledge of the system. Thus, a realistic solution is to use the nominal model of the system, which is often built by assumption, idealization, and simplicity. This approximation will result in model uncertainty, like Δf in Example 1.2. This model uncertainty is usually neglected in the existing design, which makes the model-based approaches less effective because the ignored uncertainty will affect system performance.

Thus, it is very necessary to develop some new methods to

- design the strongly nonlinear system to be robust to larger uncontrollable variations; and
- design a system to be insensitive to model uncertainty as well as variations from parameters and design variables.

1.1.2 Robust Design for Dynamic Systems

Many manufacturing systems often work under open loop, without any external control, due to some physical and economic constraints. The dynamic performance of such systems fully depends on their own design. In contrast with the design of static systems, robust design for dynamic systems must consider dynamic properties. A typical example is as follows.

Example 1.3: Dynamic system The rotor system is another basic system used in manufacturing industry. It is depicted in Figure 1.4, where a shaft carries a single disk and rotates at constant velocity Ω. Since the rotating elements are symmetrical with respect to the rotor axis and the bearings are isotropic, this system is a nonconservative system (Seyranian and Kliem, 2003; Kliem, Pommer, and Stoustrup, 1998). For simplicity, the shaft is assumed massless with the elastic coefficient $k > 0$, a single disk has mass m with the external damping d_e and the internal damping d_i, and a bearing has mass m_b with the damping d_b and the elasticity k_b.

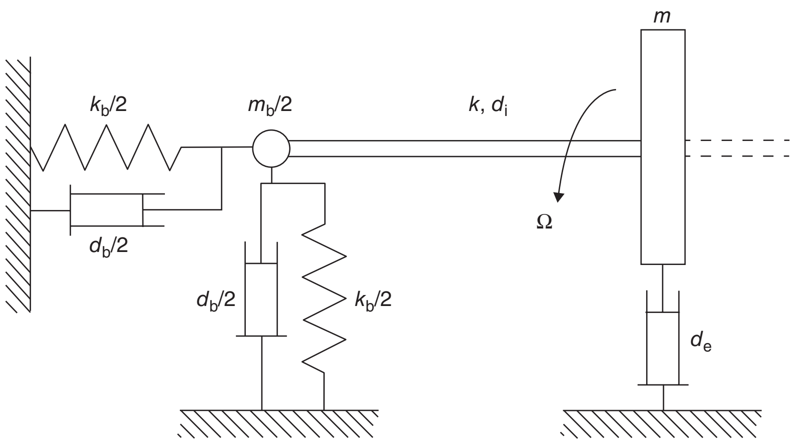

FIGURE 1.4 Physical model of a rotor system

The motion equation of the moving disk can be derived as

$$M\ddot{q} + D\dot{q} + Rq + \Delta f = 0 \tag{1.3}$$

with

$$M = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m_b & 0 & 0 \\ 0 & 0 & m & 0 \\ 0 & 0 & 0 & m_b \end{bmatrix}, \ D = \begin{bmatrix} d_e + d_i & -d_i & 0 & 0 \\ -d_i & d_b + d_i & 0 & 0 \\ 0 & 0 & d_e + d_i & -d_i \\ 0 & 0 & -d_i & d_b + d_i \end{bmatrix},$$

$$R = \begin{bmatrix} k & -k & d_i\Omega & -d_i\Omega \\ -k & k + k_b & -d_i\Omega & d_i\Omega \\ -d_i\Omega & d_i\Omega & k & -k \\ d_i\Omega & -d_i\Omega & -k & k + k_b \end{bmatrix} \Delta f = -M \begin{bmatrix} 0.01\sin(p) & 0 & 0 & 0 \\ 0 & 0.01\cos(p) & 0 & 0 \\ 0 & 0 & 0.005 & 0 \\ 0 & 0 & 0 & 0.005 \end{bmatrix} \dot{q}$$

where the parameter p has external uncertainty Δp at its nominal value p_0. The model uncertainty Δf is usually caused by unknown resistance forces. The model uncertainty is unknown and is a black box to designers.

Let

$$x = \begin{bmatrix} \dot{q} \\ q \end{bmatrix}, \ \Delta A = -M^{-1} \frac{\partial \Delta f}{\partial x} \Big|_{x=0} = \begin{bmatrix} \Delta A_{11} & 0_{4\times 4} \\ 0_{4\times 4} & 0_{4\times 4} \end{bmatrix},$$

$$\Delta A_{11} = \begin{bmatrix} 0.01\sin(p) & 0 & 0 & 0 \\ 0 & 0.01\cos(p) & 0 & 0 \\ 0 & 0 & 0.005 & 0 \\ 0 & 0 & 0 & 0.005 \end{bmatrix}$$

Since Δf and ΔA are unknown, the state–space equation for the rotor system is

$$\dot{x} = Ax \tag{1.4}$$

with the Jacobian matrix $A = A_0 + \Delta A$ and the nominal matrix $A_0 = \begin{bmatrix} -M^{-1}D & -M^{-1}R \\ I_{4\times4} & 0_{4\times4} \end{bmatrix}$.

The objective of the design is to determine design variables m and m_b to make the dynamic system stable and robust to the model uncertainty Δf and variations from parameter and design variables. ■

It is well known that the dynamic behavior of the engineering system, such as its stability, is closely related to the eigenvalues of the Jacobian matrix A. There are many studies to explicitly consider the dynamic stability using eigenvalue theory (Blanco and Bandoni, 2003; Kliem, Pommer, and Stoustrup, 1998; Mohideen, Perkins, and Pistikopoulos, 1997; Kokossis and Floudas, 1994). However, all these methods only make the real part of all eigenvalues smaller than zero, without consideration of influence from uncontrollable variations. Thus, the following problems need to be solved.

- All previous studies require the exact process model to be known without considering the influence of model uncertainty. Thus, they are difficult to apply to the partially unknown system in Example 1.3.
- Even if the process model is fully known, eigenvalue variation should be considered. Otherwise, the transient response may deviate from the desirable response (Liu and Patton, 1998). A robust transient response can only be achieved when all eigenvalue variations caused by parameter variations are small (El-Kady and Al-Ohaly, 1997). Thus, for guaranteed system stability, the eigenvalue variations should be minimal for the robust dynamic performance. Unfortunately, little attention has been paid to variations of the eigenvalues.

Thus, new design approaches should be developed not only to stabilize the system but also to minimize influence from parameter variations and model uncertainty.

1.1.3 Integration of Design and Control

As the manufacturing becomes more complex for higher quality, it becomes more difficult for traditional design and control to achieve the goal due to the hybrid nature of the process. The integration of system design and control might be needed, which is shown in the curing process in Example 1.4.

Example 1.4: Design and control of curing process The curing oven is a very important process in semiconductor packaging industry to provide a desirable temperature profile for curing epoxy resin and encapsulation molding compound that are distributed onto electronic components (Hisung and Pearson, 1997). The key requirement for high quality packaging is to maintain the uniform temperature for

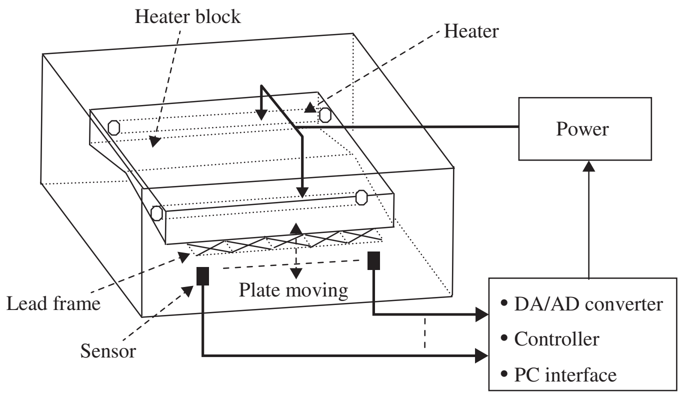

FIGURE 1.5 The studied curing system

the whole cured object and control it to follow the required temperature trajectory during the operation (Li, Deng, and Zhong, 2004).

As shown in Figure 1.5, a curing oven has a motion mechanism inside the chamber, which moves a working plate up and down to adjust the curing temperature for the IC placed on the lead frame (LF). A separate control system is also required to control the power of the heaters that are embedded in the heater block.

The finite difference method, a common modeling method for complex thermal processes, is used to model the curing process. The two-dimensional surface of the lead frame is discretized into many small zones by uniform intervals Δx and Δy, as shown in Figure 1.6. The coordinate of the zone (i, j) is $(x_i, y_j, 0)$. Each zone is assumed to have a uniform temperature, heat flux, and radiative property. Since the lead frame is thin enough, axial thermal gradient may be neglected.

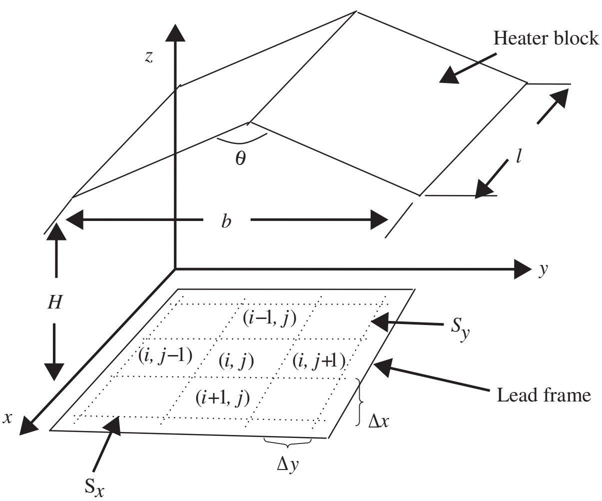

FIGURE 1.6 Geometric explanation

The heat transfer model of every zone may be described as

$$m_{i,j}c\frac{dT_{i,j}(t)}{dt} = q^{\mathrm{d}}_{i,j} + q^{\mathrm{r}}_{i,j} + q^{\mathrm{c}}_{i,j} + q^{\mathrm{wall}}_{i,j} + q^{\mathrm{dist}}_{i,j} \qquad (i = 1, \ldots, n;\ j = 1, \ldots, p) \quad (1.5)$$

where

$T_{i,j}(t)$	the temperature of the (i, j) zone at time t;
c and $m_{i,j}$	the specific heat coefficient and mass of the (i, j) zone, respectively;
$q^{\mathrm{d}}_{i,j}$, $q^{\mathrm{r}}_{i,j}$, and $q^{\mathrm{c}}_{i,j}$	heat flow rates coming into the (i, j) zone via conduction, radiation from heater block and convection from air, respectively; and
$q^{\mathrm{wall}}_{i,j}$ and $q^{\mathrm{dist}}_{i,j}$	unknown heat from the chamber wall and disturbance respectively.

According to Fourier's rule of heat conduction, heat conduction across a surface is expressed as

$$q^{\mathrm{d}}_{i,j} = kS_x\left(\frac{T_{i+1,j}(t) + T_{i-1,j}(t) - 2T_{i,j}(t)}{\Delta x}\right) + kS_y\left(\frac{T_{i,j+1}(t) + T_{i,j-1}(t) - 2T_{i,j}(t)}{\Delta y}\right) \quad (1.6)$$

where k denotes thermal conductivity, and S_x and S_y are the cross-sectional area of every zone, as shown in Figure 1.6.

The radiative heat of a zone is

$$q^{\mathrm{r}}_{i,j} = \varepsilon S_{i,j}\left\{F_{i,j}(d)u(t) - \sigma T^{4}_{i,j}(t)\right\} \quad (1.7)$$

where

ε and σ	the emissivity of the lead frame and Boltzmann constant, respectively;
U	the control variable that offers power to heaters; and
$F_{i,j}(d)$	the view factor from the (i, j) zone to the heater block; it is a function of design variables $d = [\theta, l, b, H]$, where θ, l, b are curve angle, length, and breadth of the heater block, respectively, and H is distance between the LF and the heater block as shown in Figure 1.6.

Since the heat convection has a small effect compared with the other heat flux, it can be regarded as a disturbance. Define

$$\hat{w}_{i,j}(t) = \frac{1}{m_{i,j}c}\left(q^{\mathrm{c}}_{i,j} + q^{\mathrm{wall}}_{i,j} + q^{\mathrm{dist}}_{i,j}\right) \quad (1.8)$$

Inserting Equations 1.6, 1.7, and 1.8 into Equation 1.5, the heat transfer model (Equation 1.5) may be rewritten as

$$\begin{aligned}\frac{dT_{i,j}(t)}{dt} = {} & \frac{kS_x}{m_{i,j}c}\left(\frac{T_{i+1,j}(t) + T_{i-1,j}(t) - 2T_{i,j}(t)}{\Delta x}\right) \\ & + \frac{kS_y}{m_{i,j}c}\left(\frac{T_{i,j+1}(t) + T_{i,j-1}(t) - 2T_{i,j}(t)}{\Delta y}\right) \\ & + \frac{\varepsilon S_{i,j}}{m_{i,j}c}\left\{F_{i,j}(d)u(t) - \sigma T^4_{i,j}(t)\right\} + \hat{w}_{i,j}(t)\end{aligned} \tag{1.9}$$

Obviously, the model (Equation 1.9) has strong nonlinearity and model uncertainty. Moreover, this curing process must work over a large operating region (temperature range: 20–200°C) to track the required temperature profile.

Notwithstanding the complex dynamics of the process described above, the quality of production requires a uniform temperature distribution on the whole LF. The required overall performance cannot be measured directly in production due to limited sensors used and the extra difficulty of both design and control. A joint optimization of the design variables and the controller might be needed for such a difficult task. ■

The traditional approach used for such a kind of task is the sequential method. In the sequential method, design and control are optimized separately, that is, design first then control. Design usually deals with the steady-state performance, such as economic optimality, while control deals with the transient dynamics (Sandoval, Budman, and Douglas, 2008). Thus, the sequential method often causes a poor dynamic performance since it is difficult to obtain an easily controlled process (Georgiadis et al., 2002; Meeuse and Tousain, 2002; Chawankul, 2005).

The integration of design and control can be developed to overcome the weakness of the sequential method, which is an important topic in the process industry. This integration method aims to optimize design and control simultaneously to obtain the desired performance for both design and control (Lewin, 1999; Chawankul, Budman, and Douglas, 2005). The major advantage is that system tasks, including the control task and the design task, can be shared rationally between off-line design and online control, and thus, it could be easier to obtain satisfactory design/control performance. Many integration methods have been studied in the recent decades (Mohideen, Perkins, and Pistikopoulos, 1996, 1997; Bansal et al., 2000a and 2000b; Georgiadis et al., 2002; Chawankul, 2005; Meeuse and Tousain, 2002; Lear, Barton, and Perkins, 1995; Swartz, 2004). However, there is still a long way to go due to the following unsolved problems.

- For easy integration and simplification of the controller design, a simple linear nominal model is usually used to approximate the process. This approximation is effective for a weakly nonlinear process around the operating point. However, when the strongly nonlinear process is working in a large operating region, such as Example 1.4, the large approximation error generated will make it difficult to achieve satisfactory performance.

- Little progress is achieved in the design for control, namely to obtain an easily controllable dynamic behavior through process design, before the external control is applied. If control aspects are not considered early in the design process, some complex systems may be rendered difficult to control. A good performance is always a proper balance between design and control aspects.

In general, new integration methods should be developed to overcome the above weaknesses of existing methods.

1.2 OBJECTIVES OF THE BOOK

Based on the analysis of the Section 1.1, the following major objectives are addressed in this book.

1. To develop novel approaches for the robust design of strongly nonlinear systems with large parameter variations and for minimizing the influence of model uncertainty and variations from design variables and model parameters.
2. To consider the dynamic performance, for example, stability and robustness, in system design under model uncertainty and variations from parameters and design variables.
3. To study new methods of integrated design and control, particularly design for control, for the hybrid system, enabling satisfactory performance over a large operating region.
4. To illustrate the application of the presented methods to selected equipment and process in either traditional industry or the modern IC industry.

In support of the above objectives, specific topics discussed in the book include

- a systematic overview and classification on robust analysis/design and the integration of design and control;
- development of robust static design approaches, respectively, for partially unknown systems or nonlinear systems under large parameter variations;
- development of robust dynamic design approaches for both stability and robustness of dynamic processes under parameter variations and model uncertainty;
- development of integrated design and control methods for the hybrid system working in a large operating region.

An attractive feature of this book is that several of the same examples are revisited in different chapters, with variations and enhancements. This will help readers to understand the property of different design methods and approaches.

1.3 CONTRIBUTION AND ORGANIZATION OF THE BOOK

The research fields of this book are depicted in Figure 1.7. The contributions of the book are in three main aspects: robust design for static/dynamic systems and integration of design and control. These actually contribute to the fundamentals of design for advanced manufacturing.

- First, for the static system related to the problems described in Section 1.1.1, three novel robust design approaches will be proposed: the variable sensitivity-based robust design approach for the nonlinear system (Chapter 3), the multi-domain modeling-based robust design for large parameter variation (Chapter 4), and the hybrid model/data-based robust design for both parameter variation and model uncertainty (Chapter 5).
- Second, for the dynamic system related to the problems described in Section 1.1.2, the robust eigenvalue design methods will be developed that allow both stability and robustness of the dynamic system to be maintained under parameter variations (linear system or weakly nonlinear system in Chapter 6, and nonlinear system in Chapter 7) and model uncertainty (Chapter 8).
- Finally, for the integration problem described in Section 1.1.3, two novel methods are proposed for integrating design and control for hybrid systems:
  1. An easily controlled dynamic behavior is first designed under parameter uncertainty, and then integrated with the process control through the robust pole assignment (Chapter 9).
  2. The nonlinear process is modeled with the fuzzy system to work over a large operating region, upon which design and control are integrated for overall performance and optimized with the particle swarm optimization (PSO) method (Chapter 10).

This book presents several newly developed methods for robust design and methods for integrated design and control. The book is organized as shown in Figure 1.8, where topics of chapters and their interconnection are provided for easy understanding. The contents of each chapter will also be summarized below for readers to have a quick knowledge of the whole book.

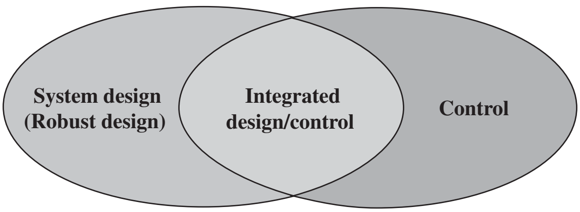

FIGURE 1.7 Research fields covered

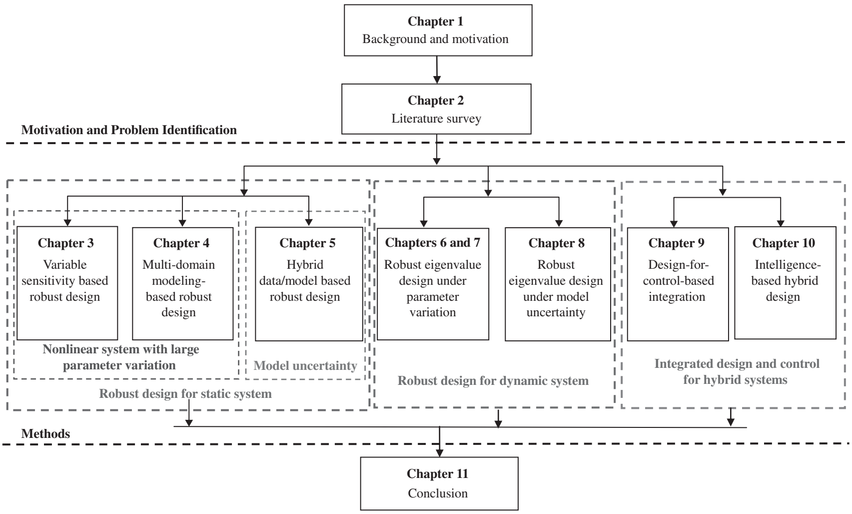

FIGURE 1.8 Organization of the book

In Chapter 2, a systematic overview and classification is presented. The characterization and quantification of uncertainty, robust analysis/design for static and dynamic systems, and the integration of design and control are briefly discussed. Various approaches are reviewed and classified with their limitations and advantages summarized for comparison. This brief overview motivates us to develop new methods for robust design and integrated design/control.

In Chapter 3, variable sensitivity based deterministic and probabilistic robust design approaches are presented for nonlinear systems (Lu, Li, and Chen, 2010; Lu and Li, 2012). A nonlinear system is first formulated using a linear structure. This linear structure will be easy to handle using well-developed robust design methods. A variable sensitivity matrix will be derived for this linear structure when the nonlinearity of the system is considered. Then, the bounds of both the variable sensitivity matrix and its singular values are calculated in a larger design region. Finally, with the variable sensitivity information incorporated, two different robust designs, one of deterministic nature and another of probabilistic nature, are developed to minimize the influence of parameter variation on the original nonlinear system. Since the proposed robust designs consider the influence of the nonlinearity, they can obtain robust performance of the nonlinear system despite uncontrollable variations.

In Chapter 4, a multi-domain modeling-based robust design approach is presented for designing a nonlinear system to be robust under large parameter variations (Lu and Huang, 2013a). Initially, a multi-domain modeling approach is used to model the nonlinear system. The model obtained has a linear structure that is easy to handle using well-developed robust design methods. Then, a robust design method is proposed to minimize the influence of large parameter variations on the performance. Since this approach integrates the merits of both the multi-domain modeling method and the robust design method to handle the influence of the system nonlinearity as well as large parameter variations, it can effectively ensure robustness of the nonlinear system even if large parameter variations exist.

In Chapter 5, two novel robust design approaches are proposed to improve the system robustness against parameter variations as well as model uncertainty (Lu and Li, 2009b; Lu, Li, and Chen, 2012). The system is first formulated as a linear structure that will be easy to handle by well-developed robust design theories. Its sensitivity matrix incorporates all model uncertainties and nonlinearities. Then, model bounds are estimated from data. Modeling the bound of model uncertainty is easier than modeling the model uncertainty itself. On this basis, the two model-based robust design methods, one deterministic and the other probabilistic, can be easily developed to minimize the influence of parameter variations on performance.

In Chapter 6, two novel robust eigenvalue design approaches are proposed to design the system to be stable and robust under parameter variations (Lu and Li, 2009a). When a linear or weakly nonlinear system has small parameter variations, a linearization model can effectively approximate the system. In this case, stability theory is first applied to obtain a set of design variables and their variation bounds. The system will be stable when design variables stay within these bounds. Then, the robust eigenvalue design is developed to make the dynamic response less sensitive to variations. Furthermore, the tolerance space of the obtained robust design will be maximized to meet the specified performance requirement for dynamic response. When the system has large parameter variations, a multi-model approach is initially developed to formulate the nonlinear relation between dynamic performance and model parameters. A stability design is then developed to guarantee the stability of the dynamic system under large uncontrollable variations. Moreover, a robust design is proposed to achieve the dynamic robustness. Finally, several examples will demonstrate and confirm the effectiveness of the proposed methods.

In Chapter 7, a novel sector nonlinearity (SN) based robust design approach is proposed to design a nonlinear system to be stable and robust. A system can be strongly nonlinear for large parameter variations, and thus difficult for traditional methods to handle. The SN method is first employed to model a nonlinear system. A stability design is then developed to ensure the nonlinear system's stability in a desirable domain under variations. Furthermore, dynamic robustness will be achieved by minimizing the sensitivity of the system eigenvalues to parameter variations. This two-loop optimization method could ensure a nonlinear dynamic system has stability and dynamic robustness under large uncontrollable parameter variation.

In Chapter 8, a novel robust design approach is proposed for stability and robustness of the nonlinear system under model uncertainty (Lu and Li, 2011). First, stability theory and perturbation theory are used to guarantee system stability under model uncertainty. Then, a new robust design is developed to make the dynamic response less sensitive to model uncertainty using matrix perturbation theory. Finally, the tolerance space of the designed variables can be maximized when there are model uncertainties and performance constraints. Thus, the proposed robust design can design the system to have the desired stability and robustness under model uncertainty and variations of design variables.

In Chapter 9, a novel method is proposed for integrated design/control of a nonlinear system with hybrid discrete/continuous variables. The controller design is simplified under the robust pole placement (Lu and Huang, 2013b). The key idea in this method is to obtain an easily controlled dynamic behavior through process design, and then integrate the merits of both design and control to ensure robust pole placement under parameter uncertainty. First, a design-for-control approach is developed to make the system controllable and have a good linear approximation. This can effectively reduce the system nonlinearity so that the system designed is easy to control. Then, a novel approach for integrated robust design and control is proposed to guarantee the stability as well as robust pole placement, which can effectively ensure satisfactory dynamic performance under parameter uncertainty.

In Chapter 10, a novel integration of design and control is proposed for the manufacturing system to work in a large operating region with the unmeasured ultimate performance (Lu, Li, Duan, and Sun, 2010; Lu, Li, and Yuan, 2010). The fuzzy modeling method is first employed to approximate the process, upon which fuzzy control rules are developed to achieve stability and robustness. Then, the process design and the control system design are integrated into a unified objective function to consider the global economic performance (high level) as well as the local dynamic performance (low level). Finally, a PSO-based global optimization method is developed to find the solution of this complex integration problem.

Chapter 11 provides conclusions and future challenges.

CHAPTER 2

OVERVIEW AND CLASSIFICATION

This chapter will provide a brief overview of robust analysis/design and its integration with control for manufacturing. It will investigate methods to account for uncertainty, to measure, evaluate, and design robustness for both static and dynamic systems, and to integrate design with control. By reviewing the strengths and weaknesses of the existing design methods, the research opportunities in this area will be discussed.

2.1 CLASSIFICATION OF UNCERTAINTY

Robust performance is one of the most important concerns in the design of any system in the manufacturing industry. This performance can be achieved by robust analysis/design. A key issue in robust analysis/design is to handle uncertainties that critically affect system performance. In general, uncertainties in robust performance design can be classified into the following categories.

- ***Noise.*** It is often stochastic and caused by variation of environmental operating conditions, for example, operating temperature, pressure, humidity, material, and so on. The robust design for this kind of uncertainty is defined as Type 1 robust design (Chen et al., 1996a).
- ***Parameter uncertainty.*** It is often caused by manufacturing errors. The model parameter of a product can be realized with certain accuracy. A design that is less

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sensitive to manufacturing tolerance will have the advantage of cost reduction without loss of performance (Beyer and Sendhoff, 2007). The robust design for this kind of uncertainty is defined as Type 2 robust design (Chen et al., 1996).

- ***Model uncertainty.*** It is often caused by approximation in system modeling. Practically, an accurate model of the system is difficult to obtain due to complex boundary conditions, unknown disturbances, or incomplete system knowledge. Thus, a realistic approximation of the system is taken as the nominal model, under educated assumptions, reasonable simplification, and idealization. This approximation will produce uncertainty in system design. The robust design for this kind of uncertainty is defined as Type 3 robust design (Choi, 2005).

On the other hand, all these uncertainties can be modeled in different ways, using deterministic-, probabilistic-, or possibility-based approaches (Beyer and Sendhoff, 2007; Choi, 2005). Different types of uncertainties are defined as follows.

- The deterministic uncertainty defines the domain in which a certain event will occur.
- The probabilistic uncertainty defines probability measure of the likelihood for a certain event to occur.
- The possibilistic uncertainty defines fuzzy measure for describing the intensive degree by which a certain event will occur.

These three different types of uncertainties are usually modeled by crisp sets, probability distributions, and fuzzy sets, respectively.

2.2 ROBUST PERFORMANCE ANALYSIS

Robust analysis only ranks the contribution of design variables to performance but cannot find the optimal solution for system robustness. These robust analysis methods can be applied at both prior- and post-design stages by ranking importance of all variables. For the prior-design situation, the ranking can help designers identify those variables with little potential impact on the response variability. Thus, the dimension of the design space can be reduced as well as the computational cost. For the post-design analysis, the ranking provides valuable information on where additional resources will be spent to further control the source of variations (Liu, Chen, and Sudjianto, 2004).

2.2.1 Interval Analysis

Interval analysis is a growing branch of computational mathematics and represents an elegant tool for operations on intervals rather than real numbers. In interval analysis, the value of a variable is replaced by a pair of numbers representing the maximum and minimum values within which the variable is expected to be. Interval arithmetic rules are then used to perform mathematical operations with the interval numbers.

In robust performance analysis, if the information about an uncertain variable in the form of probability distribution is not available, interval analysis would be the most convenient method to use. In the interval analysis, uncertain parameters are described by an upper and lower bound, and then rigorous bounds on the response are computed using interval functions and interval arithmetic. Wu and Rao (2004) integrated the interval analysis with fuzzy analysis to model the tolerances and clearances in mechanism analysis. Rao and Berke (1997) presented methods that take input parameters as interval numbers. The interval analysis was also applied to robotic mechanisms for obtaining inverse kinematics (Rao et al., 1998). Although interval analysis has been applied successfully to some problems, it is still difficult to apply to a large-scale problem because the results produced are too conservative in this complex situation (Choi, 2005).

2.2.2 Fuzzy Analysis

The possibility methods are proposed for applications where accurate statistical data are not available. Its foundation is the possibility theory. Similar to probability measures based on probability distribution functions, possibility measures can be represented by possibility distribution functions. If membership function is convexity, the possibility measure is formally equivalent to fuzzy sets (Mourelatos and Zhou, 2005). The fuzzy set methodology offers a rigorous way to quantify the membership of a design solution to the feasible solution space. In this method, an input variable in crisp nature is mapped to a fuzzy variable by a membership function $\mu(x) \in [0, 1]$. This membership describes the degree to which the input variable belongs to the feasible set. The maximum value $\mu(x) = 1$ means that x is feasible and $\mu(x) = 0$ means that x is infeasible. After the fuzzification of the input variables, the extension principle calculates the possibility distribution of the fuzzy response according to the possibility distribution of the fuzzy input variables.

The complete fuzzy response might be difficult to obtain in practice due to the overcomputation, except for simple cases involving one or two fuzzy variables. This is because the computational effort will increase exponentially with increasing number of fuzzy input variables. Therefore, numerical methods of fuzzy analysis have been proposed, such as vertex method (Penmetsa and Grandhi, 2002; Akpan, Koko, and Orisamolu, 2000) and discretization method (Akpan, Rushton, and Koko, 2002). Recently, this method was applied to modeling of tolerances and clearances in mechanism analysis (Wu and Rao, 2004) and robust optimal design to deal with epistemic uncertainty (Mourelatos and Zhou, 2005; Choi, Du, and Gorsich, 2007; Huang and Zhang, 2009; Zhou and Mourelatos, 2008). The possibility-based design was also compared with the probabilistic design for catastrophic failure under uncertainty (Nikolaidis et al., 2004). The review of this application can be found in He and Qu (2008) and Beyer and Sendhoff (2007).

2.2.3 Probabilistic Analysis

Sensitivity analysis is an important procedure in engineering design to obtain valuable information about the model behavior. Probabilistic sensitivity analysis (PSA) methods have been developed to provide insight into the probabilistic behavior of a model, which can be used to identify those nonsignificant variables and reduce the dimension space of the random design. A review about PSA was presented by (Liu, Chen, and Sudjianto, 2004). Several common methods for PSA will be reviewed as follows.

2.2.3.1 Variance-Based Methods Among the existing PSA methods, a popular category is the variance-based method, also called a global sensitivity analysis method, which aims to design the variance of a response. Reviews on different variance-based methods can be found in Chan, Saltelli, and Tarantola (1997).

These variance-based methods decompose the total variance of a model response (design performance) into variances contributed from each input variable.

$$V = \sum_{i} V_i + \sum_{i<j} V_{ij} + \cdots + V_{1,2,3,\ldots,n} \tag{2.1}$$

The first-order term V_i represents the partial variance in the response due to the effect of the input variable d_i, while the higher order term shows the interaction effects between two or more different input variables. The first-order term V_i is also called the main effect of the input variable d_i. A general sensitivity index is defined as

$$S_{i_1,\ldots,i_s} = \frac{V_{i_1,\ldots,i_s}}{V} \tag{2.2}$$

There are many approaches to obtain the above sensitivity indices. Three common approaches, namely the correlation ratio, the Fourier amplitude sensitivity test (Saltelli, Tarantola, and Chan, 1999; Chan, Saltelli, and Tarantola, 1997), and Sobol's method (Sobol′, 1993, 2001; Sobol′ and Kucherenko, 2007), will be reviewed in the following section.

1. ***Correlation ratio***

   The correlation ratio, called importance measures, evaluates variance of a conditional expectation and is defined as (Saltelli, Tarantola, and Chan, 1999; Chan, Saltelli, and Tarantola, 1997)

   $$S_i = \frac{V_{p_i}\left[E(Y\,|d_i)\right]}{V} \tag{2.3}$$

   with $V_{p_i}[E(Y|d_i)] = \int [E(Y|d_i) - E(Y)]^2 p(d_i)d(d_i)$,

   where $E(Y\,|d_i)$ denotes the expectation of Y conditional on a fixed value of d_i, and the variance is taken over all possible values of d_i. The method is only used to measure the main effect of each input on the output variance.

2. ***FAST method***

   The Fourier amplitude sensitivity test (FAST) is a procedure that has been developed for sensitivity analysis. This procedure provides a way to estimate the expected value and variance of the response and also measures the contribution of an individual input variable to the variance.

The input variable can be expressed in the following form.

$$d_i(s) = G_i(\sin(w_i s)) \tag{2.4}$$

where s is a scalar variable varying over the range $-\infty \leq s \leq +\infty$, G_i is transformation function, and w_i is frequency. According to the properties of Fourier series, an approximation of the variance of Y is given by

$$V = 2\sum_{j=1}^{\infty}\left(A_j^2 + B_j^2\right) \tag{2.5}$$

with $A_j = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(s)\cos(js)ds$, $B_j = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(s)\sin(js)ds$
where A_j and B_j are the Fourier coefficients.

The contribution to the total variance by d_i can be expressed as

$$\tilde{V}_i = 2\sum_{j=1}^{\infty}\left(A_{jw_i}^2 + B_{jw_i}^2\right) \tag{2.6}$$

Thus, the total effect index (TSI) of a random variable d_i is defined as

$$S_{Ti} = \frac{\tilde{V}_i}{V} \tag{2.7}$$

An advantage of FAST is that the evaluation of sensitivity can be carried out independently for each parameter using just one simulation because all the terms in a Fourier expansion are mutually orthogonal (Saltelli, Tarantola, and Chan, 1999; Chan, Saltelli, and Tarantola, 1997). Moreover, FAST provides a way to evaluate the variance by converting a multidimensional integral to a one-dimensional (1D) integral.

3. ***Sobol's method***

Sobol's method for variance estimation is based on an ANOVA-like decomposition of a function with increasing dimensionality (Sobol′, 1993, 2001; Sobol′ and Kucherenko, 2007). The main idea behind this method is to decompose the function into a group of increasing order terms described as below.

$$f(d) = f_0 + \sum_{i=1}^{n} f_i(d_i) + \sum_{i=1}^{n}\sum_{j=i+1}^{n} f_{ij}(d_i, d_j) + \cdots + f_{1\ldots n}(d_1, \ldots, d_n) \tag{2.8}$$

where f_0 is a constant, f_i is a function d_i only, f_{ij} is a function d_i and d_j only, and so on. Then the variance and partial variance terms can be derived as

$$V = \int f^2(d)p(d)d(d) - f_0^2 \tag{2.9a}$$

$$V_{i,\ldots,n} = \int f_{i,\ldots,n}^2(d_i, \ldots d_n)p(d_i, \ldots d_n)d(d_i) \cdots d(d_n) \tag{2.9b}$$

where $p(d)$ is the joint probability density function (PDF) of random variables d. These two equations can be evaluated by Monte Carlo methods to obtain the sensitivity index. However, this method requires a large number of samples or lengthy numerical procedures such as employing Monte Carlo or lattice samplings. Chen et al. (2005) developed an analytical technique based on meta-models in simulation-based design to improve efficiency and accuracy of Sobol's method. This kind of method can reduce the random errors associated with the sampling approach.

The major limitations of variance-based methods are as follows (Liu, Chen, and Sudjianto, 2004, 2006; Chan, Saltelli, and Tarantola, 1997).

1. Since the second moment (performance variance) is used to describe the uncertainties, a good knowledge of the distribution dispersion will be crucial for satisfactory performance.
2. The methods cannot be applied to the case where only a partial region of the distribution is counted, such as the failure region.

2.2.3.2 Sensitivity-Based Methods

1. ***Probabilistic sensitivity analysis***

   PSA evaluates the sensitivity of failure probability P_f with respect to the mean and standard deviation of the input random variable. One way is to calculate it numerically using the concept of finite difference as (Melchers, 1999):

$$S_{\theta_i} = \frac{(P_f + \Delta P_f) - P_f}{\Delta \theta_i} \tag{2.10}$$

   where θ is an uncertain measure, which is usually taken as the mean or the variance of a random variable d_i.

   The analytical equation of $\partial P_f / \partial \theta_i$ may be written as (Wu, 1994; Monanty and Wu, 2001)

$$S_{\theta_i} = \frac{\partial P_f / P_f}{\partial \theta_i / \theta_i} = E\left[\frac{\theta_i \partial p(d)}{p(d) \partial \theta_i}\right]_\Omega \tag{2.11}$$

where $p(d)$ is the joint PDF of all random variables for a failure model, and Ω denotes the failure region. Wu's sensitivity coefficients are the average impact of θ_i on the probability of failure. They are usually evaluated by sampling methods.

Equation 2.11 can be further simplified as follows if all random variables are transformed into the standard normal space (Liu, Chen, and Sudjianto, 2004):

$$S_{\mu_i} = \frac{\partial P_f / P_f}{\partial \mu_i / \mu_i} = E[u_i]_\Omega \tag{2.12a}$$

$$S_{\sigma_i} = \frac{\partial P_f / P_f}{\partial \sigma_i / \sigma_i} = E\left[u_i^2\right]_\Omega - 1 \tag{2.12b}$$

where u is a vector of standard normal random variable transformed from d. If d follows independent normal distribution, then the transformation is written as $u_i = (d_i - \mu_i)/\sigma_i$.

This method has been applied to analyze the main bearing performance of an IC engine system by Mourelatos et al. (2005). However, Wu's sensitivity coefficients are only applicable for PSA of the regional response over the failure region in order to assess the impact on the probability of failure or reliability (Liu, Chen, and Sudjianto, 2004).

2. ***Kullback–Leibler entropy-based probabilistic sensitivity analysis method***

The Kullback–Leibler (KL) entropy (Kullback and Leibler, 1951) was derived in statistics as an information measure in a random quantity y for the discrimination between the true distribution p_1 and its estimation p_0. It does not require the evaluation of the joint PDF or the conditional PDFs as needed for the mutual entropy (Krzykacz-Hausmann, 2001). The KL entropy, also called the relative entropy, is defined as

$$D_{\mathrm{KL}}(p_1 \,|p_0) = \int_{-\infty}^{\infty} p_1(y) \cdot \log \frac{p_1(y)}{p_0(y)} dy = E_{p_1}\left[\log \frac{p_1(y)}{p_0(y)}\right] \tag{2.13}$$

The KL entropy is traditionally used to measure the divergence from the true distribution p_1 to its estimation p_0. This entropy can be interpreted as the expectation of the log likelihood of a random quantity y following a PDF of $p_1(y)$. Shannon entropy or the differential entropy could be viewed as a special case of the KL entropy when p_0 is a uniform PDF.

It is assumed that a random response $y(X)$ has a PDF of p_0, where X denotes a vector of random inputs. When fixing a random input x_i at its mean value, that is, eliminating all of its uncertainty, the PDF of y changes to p_1. Therefore, the relative entropy can evaluate the total effect of x_i on the distribution of Y by measuring the difference between the two distributions: p_0 and p_1. The combined effects of the complementary of x_i, $D_{KL\sim xi}$, can be obtained by fixing all random variables except x_i and studying the change of the response PDF. The main effect of x_i is the reverse of $D_{KL\sim xi}$. By specifying the integration limits for the KL entropy computation, the method can be applied both globally and regionally.

For the globe response probabilistic sensitivity analysis (GRPSA), a KL entropy-based method measures the total and main effect indices of x_i as follows (Liu, Chen, and Sudjianto, 2004, 2006):

$$D_{\mathrm{KL}x_i}(p_1 \,|p_0) = \int_{-\infty}^{\infty} p_1(y(x_1, \dots, \bar{x}_i, \dots, x_n)) \cdot \log \frac{p_1(y(x_1, \dots, \bar{x}_i, \dots, x_n))}{p_0(y(x_1, \dots, x_i, \dots, x_n))} dy \tag{2.14a}$$

$$D_{\mathrm{KL}\sim x_i}(p_1 |p_0) = \int_{-\infty}^{\infty} p_1(y(\bar{x}_1, \dots, x_i, \dots, \bar{x}_n)) \cdot \log \frac{p_1(y(\bar{x}_1, \dots, x_i, \dots, \bar{x}_n))}{p_0(y(x_1, \dots, x_i, \dots, x_n))} dy \tag{2.14b}$$

where $\bar{x}$ means that x is taken at a given value, usually chosen at its mean value. The larger the $D_{\mathrm{KL}x_i}(p_1 |p_0)$. is, the more important x_i is. The smaller the $D_{\mathrm{KL}\sim x_i}(p_1 |p_0)$. is, the more important the main effect of x_i is. It should be noted that $D_{\mathrm{KL}\sim x_i}(p_1 |p_0)$. itself is not the main effect, but it can be used to interpret the main effect (Liu, Chen, and Sudjianto, 2004, 2006).

With simple adjustments in the formula, the proposed KL method can also be used for the regional response probabilistic sensitivity analysis (RRPSA) over a partial range of interest $[y_{\mathrm{L}}, y_{\mathrm{U}}]$ as (Liu, Chen, and Sudjianto, 2004, 2006)

$$D_{\mathrm{KL}x_i}(p_1 \,|p_0) = \int_{y_L}^{y_U} p_1(y(x_1, \dots, \bar{x}_i, \dots, x_n)) \cdot \left| \log \frac{p_1(y(x_1, \dots, \bar{x}_i, \dots, x_n))}{p_0(y(x_1, \dots, x_i, \dots, x_n))} \right| dy \tag{2.15a}$$

$$D_{\mathrm{KL}\sim x_i}(p_1 \,|p_0) = \int_{y_L}^{y_U} p_1(y(\bar{x}_1, \dots, x_i, \dots, \bar{x}_n)) \cdot \left| \log \frac{p_1(y(\bar{x}_1, \dots, x_i, \dots, \bar{x}_n))}{p_0(y(x_1, \dots, x_i, \dots, x_n))} \right| dy \tag{2.15b}$$

Over the entire range of a response distribution, the effect of a random variable is measured by its impact on the whole distribution of response. Over a specific region, the effect of a random variable is indicated by its impact on the distribution of the response within range. Obviously, the KL based method gives a more complete measure of the effect of a random variable than the variance-based measure. It should be noted that the KL methods can only give a relative importance ranking of random variables. The absolute values of the KL measures themselves are hard to interpret.

Unlike a true metric, there is still no method to normalize the KL values. The PDFs in the integral are usually obtained by sampling-based estimations. The integral can be computed by numerical methods (Liu, Chen, and Sudjianto, 2004, 2006).

2.3 ROBUST DESIGN

The fundamental principle in robust design is to improve the quality of a product by minimizing the sensitivity of the performance to uncontrollable variations. The robust design framework is shown in Figure 2.1.

The robust design methods are mainly classified into three categories (Choi, 2005): Type I robust design, Type II robust design, and Type III robust design. These methods will be introduced in detail.

Type I robust design was proposed by Taguchi. This kind of robust design is used to design systems that satisfy a set of performance requirement targets despite variations in noise factors, which are uncertain, uncontrollable, independent system parameters. The experiments are usually arranged first and conducted using the design of experiment (DoE). After collecting the experiment data, the Taguchi method is then used to identify control factor (design variable) values that satisfy a set of performance requirement targets despite variation in noise factors.

Type II robust design was presented by Chen and coauthors (Chen et al., 1996). This kind of robust design is to design a system robust to the uncertainty in control factors. Similar to noise factors, control factors are also in parametric forms that can be measured and characterized as continuous numbers with or without probability distribution (Choi, 2005). Control factors are usually derived from the characterized parameters in system models that relate to system performances, including geometric information, mass, electrical, mechanical, or chemical inputs, amounts of constituents in materials, and so on. Type II robust design identifies the control factors (design variables) to satisfy a set of performance requirement targets despite variation in control and noise factors. However, it does not pay attention to model uncertainty.

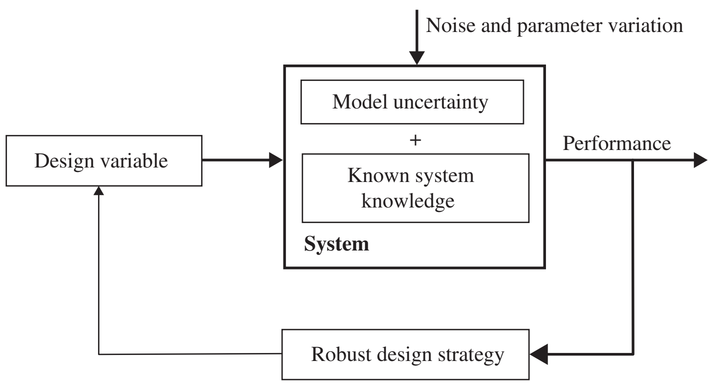

FIGURE 2.1 Robust design framework

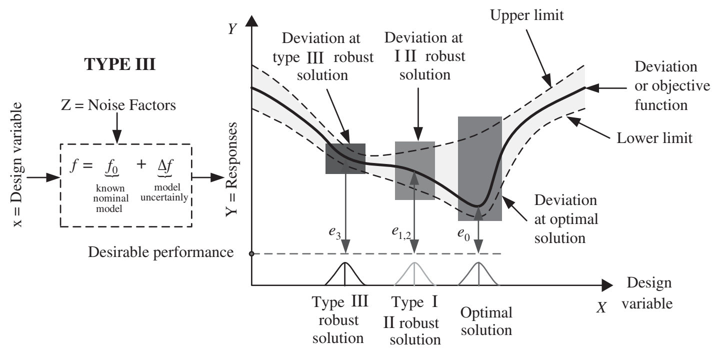

FIGURE 2.2 Type III robust design

Type III robust design is to design a system not only robust to Type I and II uncertainties but also robust to model uncertainty that can be defined as Type III uncertainty. The concept of this kind of robust design as presented by Choi (2005) is illustrated in Figure 2.2. In this figure, the same objective function curve is employed to show the differences among the optimal solutions, Type I and II robust solutions, and Type III robust solution. A deviation (or objective) function, which represents the system's response, is illustrated as a solid curve. In addition, two dotted curves are added around the objective function, representing uncertainty limits, due to the nonparametric variability, unconfigured variability, and model parameter uncertainty. Considering not only the objective function but also the two uncertainty limits, the optimal solution, and the robust solution that only considers Type I and II uncertainties will have larger performance deviations than Type III robust solution that has taken into account all Type I, II, and III uncertainties. For Type III robust design, Choi and his study group (Choi, 2005) presented a response surface method to obtain this model uncertainty, upon which a robust solution can be derived.

In past decades, much effort has been dedicated to these robust designs. The systems can be classified into the static system that does not vary over time and the dynamic system that varies over time. Thus, the robust design methods can also be classified into two categories: robust design for the static system and robust design for the dynamic system. All these works will be reviewed in the following sections.

2.3.1 Robust Design for Static Systems

This robust design minimizes the effect of variations, including parameter variation, noise and variation of design variable, on the static performance. The general robust design problem is described as

$$Y = f(d, p) \tag{2.16}$$

where $Y = [y_1 \ \cdots \ y_m]^T$ represents static performance vector, $d = [d_1 \ \cdots \ d_n]^T$ is design variable vector, $p = [p_1 \ \cdots \ p_l]^T$ is design parameter vector, and $f(\cdot) = [f_1(\cdot) \ \cdots \ f_m(\cdot)]^T$ represents the system model.

Work on this topic can be classified into two categories (Li, Azarm, and Boyars, 2006): deterministic robust design and probabilistic robust design.

2.3.1.1 Deterministic Robust Design The objective of deterministic robust design is to minimize the worst case of the performance caused by deterministic uncertainties. Its advantage is that it does not require the probabilistic distribution of parameters. It mainly includes Euclidean norm method, conditional number method, the sensitivity region measure method, and the robust space search method.

1. ***Euclidean norm method and conditional number method***

Taking Taylor series expansion of Y at the nominal parameter values p_0 and neglecting its high order term, the following equation is obtained.

$$Y = f(d, p_0) + J\Delta p \tag{2.17}$$

with the sensitivity matrix $J = \frac{\partial f(d,p)}{\partial p}|_{p=p_0}$

where p_0 is nominal parameter, and parameter variation $\Delta p = p - pc_0$.

Define the performance variation ΔY as

$$\Delta Y = Y - f(d, p_0) \tag{2.18}$$

Then, the sum of the performance variation will be represented as

$$(\Delta Y)^T \Delta Y = (\Delta p)^T J^T J \Delta p \tag{2.19}$$

For designers, parameter variation Δp cannot be controlled, and only the sensitivity matrix J can be designed by selecting a suitable design variable d. Then, the singular value decomposition is used to decompose the sensitivity matrix J

$$J^T J = \zeta^T \sigma \zeta \tag{2.20}$$

where σ_i is the singular value of the sensitivity matrix J, and the corresponding orthogonal eigenvector is denoted as ζ_i, which is one element of $\zeta = [\zeta_1 \ \cdots \ \zeta_n]$.

Inserting Equation 2.20 into Equation 2.19, the performance variations ΔY can be expressed as follows (Rajagopalan and Cutkosky, 2003; Caro, Bennis, and Wenger, 2005; Zhu and Ting, 2001)

$$\|\Delta Y\|_2^2 = \sum_{i=1}^{n} \sigma_i (v_i)^2 \tag{2.21}$$

with $[v_1, \ldots, v_n]^T = \zeta^T \Delta p$.

Obviously, from Equation 2.21, the performance in the m-dimensional space is a hyperellipsoid. Its 2D projection is depicted in Figure 2.3.

The ideal solution requires that all principal axes of this hyperellipsoid be as long as possible, which would be difficult to realize in practice. Two alternative

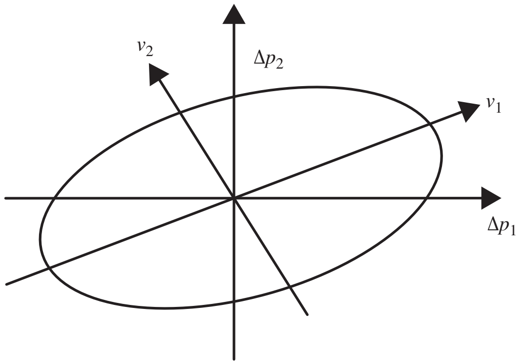

FIGURE 2.3 A 2D sensitivity ellipsoid

measurement methods are the Euclidean norm method and the conditional number method.

The Euclidean norm method is to maximize the length of the shortest principal axis (Caro, Bennis, and Wenger, 2005; Zhu and Ting, 2001). Since the shortest principal axis corresponds to the largest singular value $\sigma_{\max}$, if the largest singular value $\sigma_{\max}$ is minimized, then the shortest principal axis will be maximized. So the robust design in the Euclidean norm method is to minimize $\sigma_{\max}$.

The condition number method (Caro, Bennis, and Wenger, 2005; Ting and Long, 1996) is to minimize the ratio of the shortest principal axis to the longest principal axis. The longest principal axis corresponds to the smallest singular value $\sigma_{\min}$ and the shortest principal axis corresponds to the largest singular value $\sigma_{\max}$. If the condition number $\sigma_{\max}/\sigma_{\min}$ is minimized, then the ratio between the longest principal axis and the shortest principal axis will have the relatively smaller value. Thus, the condition number method is to minimize the condition number $\sigma_{\max}/\sigma_{\min}$ of the sensitivity matrix J.

Caro, Bennis, and Wenger (2005) have compared these two methods on the damper design. The results have shown that the Euclidean norm method is more suitable as the robust index than the condition number method. Although these two design methods have achieved many successful applications, still they have the following limitations.

- The gradient information of variables has to be known. However, this gradient information is often difficult to obtain in practical applications.
- Since the design relies on the linear model obtained from the Taylor series expansion, the design is only applicable to the weakly nonlinear system.
- The requirement for an accurate system model could make the design ineffective for the partially unknown system.

2. ***Sensitivity region measure method***

As described in Figure 2.4, the sensitivity region measure (Gunawan and Azarm, 2004) first projects the desirable performance space into the parameter

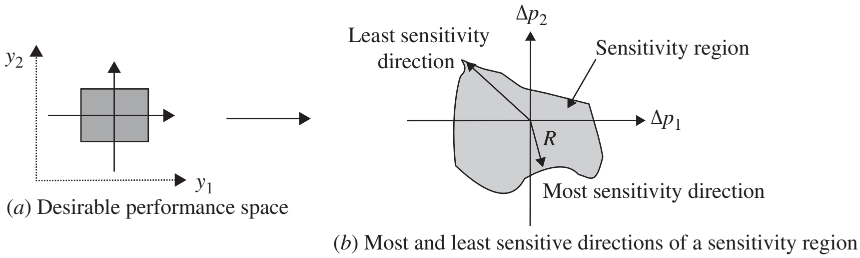

FIGURE 2.4 Sensitivity region and robust measure

space using the system model, upon which the sensitivity region is constructed. Then the most sensitivity direction, which means the minimal distance R from the bound of the sensitivity region to the central point, is found and used as a robust measure. The larger the distance R is, the more robust the system is. Thus, the robust design can be obtained by minimizing the distance R. This method has been applied to the robust optimization problem (Gunawan and Azarm, 2004, 2005a, 2005b).

As described in Figure 2.5, another sensitivity region measure is the objective sensitivity region measure (Li, Azarm, and Boyars, 2006). It first projects the model parameter space into the desirable performance space, upon which the objective sensitivity region is constructed. Then, the maximal performance variation, which is represented by the maximal distance R from the bound of the objective sensitivity region to its central point, is found and used as a robust measure. The smaller the distance R is, the more robust the system is. Thus, the robust design can be obtained by minimizing the distance R. The comparison between the sensitivity region measure method and Gunawan's method (Gunawan and Azarm, 2004) is carried out. The results have shown that Gunawan's approach is not applicable if the objective and constraint functions are discontinuous with respect to parameter variations. This method (Li, Azarm, and Boyars, 2006) does not require the objective functions to be continuous.

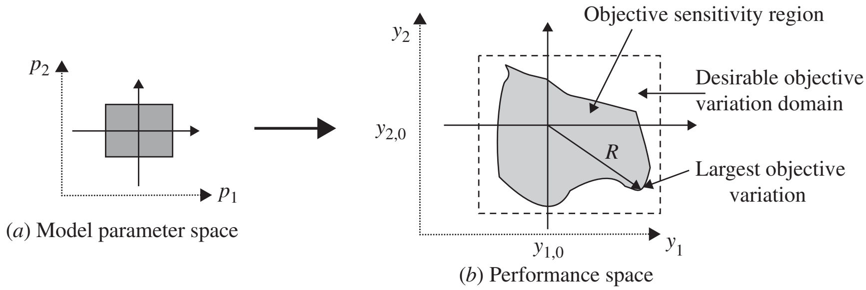

FIGURE 2.5 Objective sensitivity region and robust measure

These robust design methods have the following advantages.

- They do not need to know the gradient information and a presumed probability distribution of uncontrollable parameters.
- They are applicable to these nondifferentiable and/or discontinuous design problems.

These robust design methods also have the following limitations.

- The projection will be difficult for the high dimensional system, which requires a high computational cost.
- They need to know the exact system model for obtaining the robust performance index.

3. ***Robust space search method***

In the robust space search method (Parkinson, 2000), the tolerance space for every candidate design is first defined as $l_i \leq d_i \leq u_i$ with the given tolerance limits l_i and u_i. Then, the maximum and minimum of the performance can be figured out if design variables vary in their tolerance space. The smaller the difference between the maximum and minimum of the performance is, the more robust the design is. Obviously, this method is simple and direct. Thus, the robust design can be figured out by solving the following optimization problem

$$\min_{d_0}(y_U - y_D) \tag{2.22}$$

where y_U and y_D are the maximum and minimum of the performance y in the tolerance space, respectively.

However, they have some limitations as follows.

- If the tolerances of the parameters are unknown, this method will not work.
- The exact system model needs to be available and large computation may be required for the high dimensional system.

2.3.1.2 Probabilistic Robust Design Since the probabilistic robust design makes use of probabilistic information of the parameter, it will be less conservative than the deterministic robust design. Its main objective is to minimize the mean or variance of the performance. In this section, several probabilistic robust designs will be reviewed.

1. ***Monte Carlo simulation***

Monte Carlo simulation method is a class of computational algorithms for performance design. First, the system input is randomly sampled according to its PDF that is known or assumed beforehand. Then, an experiment is executed under the generated samples and the performance data are collected. Finally, the statistic method is used to estimate its PDF from the collected data. It is well known that a game of chance is constructed from probabilistic properties, from which the required result is deduced (Melchers, 1999).

Monte Carlo method tends to be used when it is unfeasible or impossible to compute an exact result using a deterministic algorithm (Hubbard, 2007), and is especially useful in studying systems with a large number of coupled variables, such as fluids, disordered materials, strongly coupled solids, and cellular structures. However, it has slow convergence and may require thousands or millions of data samples to obtain sufficient accuracy. Moreover, it requires the exact system model. These limitations and disadvantages limit its applications.

2. ***Taguchi method***

Taguchi robust design is an approach to identify design variables that satisfy a set of performance requirements despite variation in noise factor (Ross, 1988; Taguchi, 1987, 1993). This method can improve the quality of a product by minimizing the effect of variations without eliminating their causes. Taguchi robust design includes experimental design (orthogonal array), quality loss function, and signal-to-noise ratio.

To measure quality, Taguchi defines a quality loss function. This quality loss function as shown in Figure 2.6 is a continuous function that is defined in terms of the deviation of a design parameter from an ideal or target value.

$$L(y) = K(y - T)^2 \tag{2.23}$$

where L is the loss, y is the performance, T is the target value, and K is a cost coefficient. This loss function demonstrates the loss degree of the product when the performance deviates from the target value. The main objective of this function is to let a product engineer understand what the target value is. This loss function can approximate the behavior of loss in many instances (Byrne and Taguchi, 1986).

Taguchi robust design approach for the parameter design starts from experimental design. In this robust design, the orthogonal array is used as the experimental design. Control factors and noise factors reside in an inner array and an outer array, respectively. The experimental results obtained by all combinations of control factors and noise factors are recorded. Then, Taguchi proposed a signal-to-noise ratio for measuring sensitivity analysis of response to variation of noise factors. Based on the signal-to-noise ratio, the robust design solution is obtained.

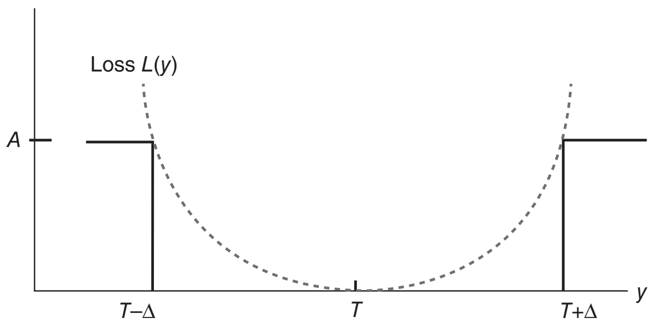

FIGURE 2.6 Taguchi's loss function

Although Taguchi's method has achieved great success, there are certain assumptions and limitations associated with this method, which have been criticized by the statistical community (Box, 1988; Nair, 1992; Tsui, 1992). Use of the Taguchi method will not yield an accurate solution for design problems that embody highly nonlinear behavior (Chen et al., 1996). Many of Taguchi's statistical methods, for example, orthogonal arrays, linear graphs, and accumulation analysis, are not statistically efficient (Tsui, 1992). Moreover, the Taguchi's method cannot handle the variations of design variables. Thus it is only suitable for Type 1 robust design.

3. ***Response surface method***

The response surface method is a fully data-based modeling method and usually uses the first- or second-order polynomial to approximate the system. Statisticians (Box, 1988; Tsui, 1992; Engl, 1992) suggested modeling the mean response and variance directly with statistical data transformation instead of the signal-to-noise ratio. Vining and Myers (1990) proposed a dual response approach, minimizing variability while keeping the mean on target. Nelder and Lee (1991) and Myers, Khuri, and Vining (1992) suggested applying generalized linear models (GLM) to robust design. Grego (1993) used a GLM for the variance of the response from replicated classical experiments. A robust design procedure, which integrated the response surface methodology with the compromise decision support problem, was developed by Chen et al. (1996) to overcome the limitations of Taguchi's methods. Xu and Albin (2003) captured statistical error bounds for a response surface model constructed with experimental data during a simultaneous confidence interval. The uncertainty in the response surface model was statistically estimated for the robust optimization. Although the response surface method has obtained many successful applications, its full reliance on data without consideration of system information may lose the modeling accuracy.

4. ***First- and second-order moment methods***

These moment methods, which are derived by Taylor series expansion, are the most widely used methods to estimate the probabilistic information of performance. Taylor series expansion is employed to estimate response variance based on input parameter variances as

$$\sigma_{\text{first\_order}}(Y) = \left(\left.\frac{\partial f}{\partial p}\right|_{\mu_p} \right)^2 \sigma(p) \tag{2.24}$$

$$\sigma_{\text{second\_order}}(Y) = \left(\left.\frac{\partial f}{\partial p}\right|_{\mu_p} \right)^2 \sigma(p) + \frac{1}{2}\left(\sigma(p) \left.\frac{\partial^2 f}{\partial p^2}\right|_{\mu_p} \right)^2 \tag{2.25}$$

where σ and μ denote variance and mean, respectively.

In the first-order method, the variance of the output is equal to the variance of the input parameters multiplied by the square of the first sensitivity derivative evaluated at the mean value of the input, as indicated in Equation 2.24. This method is simple but with relatively low precision because it only takes the first-order Taylor series expansion. To improve the estimation, the second-order method incorporates the higher-order term (second-order term) in the analytical model, as shown in Equation 2.25.

These moment methods are very simple and convenient. Therefore, these methods are widely used as approximate methods for response estimation (Rajapopalan and Cutkosky, 2003; Du and Chen, 2000; Al-Widyan and Angeles, 2005). The robust design for these approximate models is to minimize the sensitivity derivative so that the variance of the response will be minimized. Although these methods are good for Gaussian probability distribution, they are very hard to apply to other types of probability distributions in input parameters (Choi, 2005). Also, the result could be inaccurate when the system has strong nonlinearity due to the approximation. Thus, the method highly relies on the accurate system model obtained.

5. ***Probabilistic performance design***

The probabilistic performance design mainly includes Suh's information content, design preference index, and design capability indices, which will be reviewed below.

- ***Suh's information content***

Suh (1990, 2005) proposed two well-known design axioms,

Axiom 1: The independence axiom

Maintain the independence of performance requirements.

Axiom 2: The information axiom

Minimize the information content of the design (uncoupled design with less information).

The information axiom is used to evaluate the quality of design so that an appropriate design can be chosen from available design alternatives. Information content is defined in terms of entropy, which is expressed as the logarithm of the inverse of the probability of success p as

$$I = \log_2 \frac{1}{p} \tag{2.26}$$

As shown in Figure 2.7, the design range is the desirable range for meeting the performances, the system range is the performance of a candidate system,

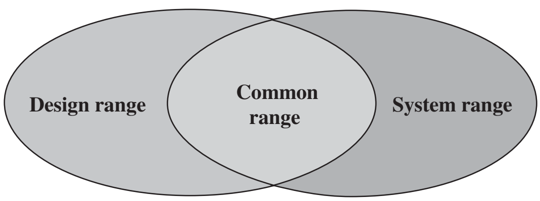

FIGURE 2.7 Design range, common range, and system range

and the common range is the overlapping region between the design range and the system range.

In case of uniform probability distribution of design range, the information content can be expressed as

$$I = \log_2 \left(\frac{system\ range}{common\ range} \right) \tag{2.27}$$

According to the information axiom, a design candidate that has the minimal information content should be selected based on the calculated probability of success.

- ***Design preference index***

  Design preference index (DPI) is defined as (Chen and Yuan, 1999; Wallace, Jakiela, and Flowers, 1996)

$$\mathrm{DPI} = E(p(Y)) = \int_{\bar{Y}-\Delta Y}^{\bar{Y}+\Delta Y} p(Y)g(Y)dY \tag{2.28}$$

where the preference function $p(Y)$ is a function defining the relationship between the degree of desirability p and the level of performance Y. It is in the range of zero and one, with "one" representing the full preference, and "zero" representing no preference, that is, unacceptable. The function $g(Y)$ is the PDF.

This index is used as a measure to evaluate the goodness of the performance. When the DPI of the design is one, this design can fully meet the design requirements under uncertainty. Otherwise, the design requirements are difficult to fully satisfy. It is desired to maximize DPI as close as possible to one.

- ***Design capability indices***

  The six standard deviation range ($\pm 3\sigma$) is a common measure of process capability, which compares variation of a process to the customer specifications as below

$$C_p = \frac{\mathrm{USL} - \mathrm{LSL}}{6\sigma} \tag{2.29}$$

where USL and LSL are the upper specification limit and the lower specification limit of the performance, respectively, and σ is its standard deviation. If C_p is less than one, the process variation exceeds the specification limits, and the performance cannot satisfy the requirement. Conversely, if C_p is greater than one, the process variation is less than the specification limits, and the performance can satisfy the requirement. If C_p is equal to one, the process just meets customer specifications, a minimum of 0.3% defectives will be made (provided that the process mean is centered on the target value).

Although C_p is a measurement of the spread of the process in relation to specification width, it does not measure how well the mean of the process is close to the target.

The concept of the above capability index was extended to consider the approximate degree between the mean of the system and the target value (Chen et al., 1996b, 1999). They proposed the design capability indices (DCI) as metrics for system robustness. The indices C_{du} under the upper requirement limit, C_{dl} under the lower requirement limit, and C_{dk} under the upper and lower requirement limits are defined as

$$C_{dl} = \frac{\mu - \mathrm{LRL}}{3\sigma}, \; C_{du} = \frac{\mathrm{URL} - \mu}{3\sigma}, \; C_{dk} = \min(C_{dl}, C_{du}) \qquad (2.30)$$

where URL and LRL are upper and lower requirement limits.

When the index is smaller than unity, it means that the system performance can be outside of the system requirement range. If the index is equal to or greater than unity, then the design will meet the requirement satisfactorily. Therefore, a designer's objective is to force the index to be unity so that the larger portion of performance deviation falls into the range of design requirements. Forcing the index to be unity is achieved by reducing performance deviation and/or locating the mean of performance to the center of the requirement range. The applications of these indices may be found in Chen et al. (1999) and Xiao et al. (2005).

Remark: These probabilistic performance designs offer measures for robust performance with a clear physical meaning. The geometrical interpretations of these methods make it easy to understand and apply. However, in these probabilistic performance designs, the system range, the variance, and mean of the system must be known beforehand through other design methods, such as Monte Carlo simulation and moment methods.

2.3.2 Robust Design for Dynamic Systems

In this kind of robust design, the uncertainties are time varying and caused by variations in either external variables (such as disturbance, etc.) or internal process parameters (such as heat/mass coefficients, kinetic constant, etc.) (Dimitriadis and Pistikopoulos, 1995).

2.3.2.1 Robust Stability Design The system stability relies on its eigenvalues. If all eigenvalues are on the left side of the complex plane, the system is exponentially stable; if any eigenvalue stays on the right side of the complex plane, the system will be unstable. Practically, the system eigenvalues are dependent on the parameters that could include the operating environment variables, model parameters, and manufacturing operations. Since variations of these parameters are unavoidable, increasing effort has been devoted to consider the effect of variations on system stability at the process design stage, which may include

- the Lyapunov's stability matrix equality that can guarantee system stability under uncertainty and disturbance (Blanco and Bandoni, 2003);
- the matrix measures based robust stability criteria that can provide a single upper bound for all eigenvalues (Mohideen, Perkins, and Pistikopoulos, 1997; Kokossis and Floudas, 1994); and
- the manifold-based robust stability design that figures out the bound of parameter variations, which guarantees all eigenvalues of the process to be smaller than zero when parameter variations are limited in this bound (Monnigmann and Marquardt, 2003, 2005; Grosch, Monnigmann, and Marquardt, 2008).

However, they still have some limitations:

- Both the positions and variations of the eigenvalues, which closely relate to stability and robustness, are of vital importance for the dynamic system. However, the existing stability designs only consider the system stability with little consideration of eigenvalue variations. This may cause deviation of the transient response from the desired performance (Liu and Patton, 1998).
- The reliance on the exact system model will make the design inapplicable to the partially unknown system.

2.3.2.2 Feasibility Design The flexibility, the ability to maintain feasible operation over a range of uncertain conditions, is a vital important character for the system operation and should be explicitly incorporated into the design objectives. The flexibility problem generally consists of two tasks that are complementary to each other (Dimitriadis and Pistikopoulos, 1995).

- The first task is to determine whether a given design can feasibly operate over the range of uncertainty considered. This problem is known as the feasibility problem or flexibility test problem.
- The second task is to calculate a measure to quantify the ability of the design to operate in the presence of uncertainty. This is known as the flexibility index problem and is usually tackled by establishing the maximum parameter range over which the design will be feasible.

The flexibility measure is often used to select the suitable design by comparing different design alternatives (Grossmann and Straub, 1991; Dimitriadis and Pistikopoulos, 1995), which can be summarized as follows.

- ***The deterministic flexibility index.*** This index was defined by Swaney and Grossmann (1985a) for measuring the flexibility of processes in steady state where uncertain parameters are described by bound of the operation range. Many methods are used to figure out this flexibility index, such as the vertex enumeration search method (Swaney and Grossmann, 1985b), the implicit enumeration procedure (Kabatek and Swaney, 1992), the active set strategy (Grossmann and Floudas, 1987), the sensitivity method (Varvarezos, Grossmann, and Biegler, 1995), and the KS function method (Raspanti, Bandoni, and Biegler, 2000). It is also extended to the analysis of dynamic system under time-varying uncertainties (Dimitriadis and Pistikopoulos, 1995; Sakizlis, Perkins, and Pistikopoulos, 2003; Bansal et al., 2000a & b; Malcolm et al., 2007).

- ***The stochastic flexibility index.*** It is a metric for quantifying the ability of a process to maintain feasible operations in the presence of stochastic uncertainties. For linear steady-state models, the stochastic flexibility index is defined as a probabilistic metric, upon which a given design will operate feasibly in the presence of uncertainties described by Gaussian probability distribution functions (Pistikopoulos and Mazzuchi, 1990; Straub and Grossmann, 1990). Extensions have also been reported to nonlinear steady-state systems as well as linear dynamic systems (Straub and Grossmann, 1993; Pistikopoulos and Ierapetritou, 1995; Bansal, Perkins, and Pistikopoulos, 1998). A parametric programming framework was presented for the solution of flexibility analysis and the design optimization problems in linear processes with both deterministic parameters and stochastic parameters (Bansal, Perkins, and Piskopoulos, 2000, 2002).
- ***The distance-based flexibility index.*** The nominal operating point is moved away from the constraints (called back-off point) so that the constraints would not become active when disturbance enters the system (Georgakis et al., 2003; Bandoni et al., 1994). The bigger the distances between the nominal operating point and the constraint boundaries are, the better the system flexibility is. Its optimization arithmetic consists of two stages called outer and inner loops. Each loop is formulated as a semi-infinite optimization problem. Outer loop figures out the best operating condition for a given set of disturbance. In the inner loop, the feasibility of the operating conditions obtained in the last outer loop is tested. Repeat the outer and inner loops, and then a desirable design will be achieved. Bandoni et al. (1994) and Bahri et al. (1995, 1996) applied this method to analyze the steady-state flexibility of a chemical plant. This method was also extended to the analysis of dynamic systems (Bahri et al., 1997; Figueroa et al., 1996).
- ***The ratio of the feasible space size to the overall operation space*** (Lai and Hui, 2007, 2008).

2.3.2.3 Controllability Design The controllability design is to design the system to have a good dynamic behavior, so that it can be easily controlled.

1. ***Resiliency index***

   Resiliency measures the degree to which a system can meet its design objectives despite external disturbance and uncertainty. Because adequate process resiliency is a necessary part of the optimal process design, it is desirable to consider process resiliency assessment when determining the process structure and establishing the operating range. The ratio of the control input u to the disturbance w is often regarded as the resiliency index (Skogestad and Morari, 1987; Wettz and Lewin, 1996; Lewin, 1996)

$$\gamma_I = \frac{||u||_2}{||w||_2} \tag{2.31}$$

This resiliency index measures the effect of the disturbance on the control input u. Cao, Rossiter, and Owens (1997) applied it to select the control inputs. Solovyev and Lewin (2003) extended this steady-state resiliency index to the nonlinear system for the analysis of the designed process.

2. ***Condition number***

The condition number of a matrix is defined as the ratio between the largest singular value $\overline{\sigma}(G)$ and smallest nonzero singular value $\underline{\sigma}(G)$ of the transfer function $G(s)$

$$\gamma_C = \overline{\sigma}(G)/\underline{\sigma}(G) \tag{2.32}$$

The condition number provides a direct measure of the directionality of the system. A large condition number indicates that the system gain changes significantly along a certain input direction, which tells that the performance strongly relies on the specific type of perturbation (Bezzo, Varrasso, and Barolo, 2004; Skogestad and Havre, 1996; Chen, Freudenberg, and Nett, 1994). Thus, a large condition number does imply that the system is ill conditioned and sensitive to “unstructured” input uncertainty.

3. ***Disturbance condition number***

The following linear system is considered

$$y(s) = G(s)u(s) + G_d(s)w(s) \tag{2.33}$$

where y is output, u is control input, and w is disturbance, and G and G_d are transform functions.

The disturbance condition number is defined as (Skogestad and Morari, 1987)

$$\gamma_d = \frac{\|G^{-1}G_d d\|_2}{\|G_d d\|_2}\bar{\sigma}(G) \tag{2.34}$$

The disturbance condition number is a measure of the input magnitude to reject the disturbance along a certain direction, where the control effort is minimal compared with other directions. Input sets yielding a small γ_d are the most effective for disturbance rejection (Wal and Jager, 2001).

4. ***Operability design***

The operability measure can quantify the inherent ability of the process to move from one steady state to another and to reject the expected disturbances. The operability index is defined as (Vinson and Georgakis, 1998, 2000)

$$\mathrm{OI} = \frac{R[\mathrm{AIS} \cap \mathrm{DIS}]}{R[\mathrm{DIS}]} \tag{2.35}$$

where R is a measurement function calculating the size of the corresponding space, for example, in two dimensions it represents the area, and in three dimensions it represents the volume; AIS is the available input space, in which the inputs of the process are able to change; DIS is defined as the set of input values on which the entire desired output space can be reached, which is actually the operating window for the process outputs.

This index can effectively capture the inherent operability of continuous processes. Its geometrical interpretation makes it easy to understand and apply. This index has been applied to analyze the linear steady-state system (Vinson and Georgakis, 1998, 2000), the nonlinear system (Subramanian and Georgakis, 2000, 2001), and the dynamic system (Uztürk and Georgakis, 1998, 2002; Subramanian, Uztürk, and Georgakis, 2001). All these works have been properly surveyed by Georgakis et al. (2003).

2.4 INTEGRATION OF DESIGN AND CONTROL

Integrated process design and control will involve continuous control action and discrete decision. A unified framework for this integration is actually a hybrid discrete/continuous system that will be closely linked to the process topology, equipment design specifications, control structure configuration, and controller design (Seferlis and Georgiadis, 2004; Seferlis and Grievink, 2004; Ross, 1999). Both continuous and discrete variables involved in the integration will be determined through the optimization of multi-objective functions, subject to the static and dynamic performance requirement under uncertainty.

2.4.1 Control Structure Design

Most available control theories assume the control structure to be given beforehand. For a complex system, it may not be easy to answer basic questions like, which variable should be selected to control, and which inputs should be manipulated (Skogestad and Postlethwaite, 2005).

2.4.1.1 Pairs of Input and Output The main method for selecting pairs of input and output is the relative gain matrix (RGA), which was originally proposed by Bristol (1966). The objective of RGA is to provide a measure of interactions for multivariable square systems (Skogestad and Morari, 1987; Yu and Luben, 1986). The RGA is defined as

$$\Lambda(G) = G(G^{+})^{T} \tag{2.36}$$

where G^{+} is the pseudo-inverse of the system transform function G.

This RGA was used to select the pairs of input and output for feedback control (Skogestad and Postlethwaite, 2005; Kariwala, Forbes, and Meadows, 2003). It was also proven that, if the plant has large RGA elements, it will be difficult to achieve good tracking with feedforward control due to its strong sensitivity to uncertainties from diagonal inputs (Skogestad and Havre, 1996).

The concept of RGA was applied to design block relative gain, so that partially decentralized control systems can be handled (Manousiouthakis, Savage, and Arkun, 1986). It was also extended to nonsquare multivariable systems for selection of square subsystems from the nonsquare system (Chang and Yu, 1990). A dynamic relative gain was proposed by Avoy et al. (2003) and a tight bound was calculated on the worst case of relative gains under norm-bounded uncertain systems (Kariwala and Skogestad, 2006).

2.4.1.2 Selection of Controllable Variables In order to obtain an optimal performance, you need to know what to control. Although it is not widely acknowledged by control theorists, selection of right variables to control will be crucial to uncertainty suppression during the entire operation (Alstad and Skogestad, 2007). Many studies have been reported on this aspect, for example, the singular value-based method and the local method (Halvorsen et al., 2003), the null space method (Alstad and Skogestad, 2007), the average loss method (Kariwala, Cao, and Janardhanan, 2008), and so on. The selection of controllable variables has been applied to indirect control (Hori, Skogestad, and Alstad, 2005).

Some studies have been reported on controller selection according to process characteristics. Hernjak et.al (2004) selected the control system according to the process characteristics, such as degree of nonlinearity, dynamic character, and degree of interaction. Seferlis and Grievink (2001 & 2004) suggested screening the process design and control structure based on economic and static controllability criteria.

Remark: In the early stage of integration development, the attention is mainly paid to screen alternative designs according to some controllability indices. These methods can obtain the desirable steady-state performance by rejecting the poor performance in an early design stage. However, as presented by Meeuse and Tousain (2002), the indices are often calculated based on steady-state data only. The relation between the indices and the closed-loop performance is not fully studied. Thus, the role of controller is less considered in the process design.

2.4.2 Control Method

Controller design will be a critical issue in the integration of design and control. The economic performance of the designed process should be considered together with other design constraints. Many control methods have been studied in this kind of integration, such as PI or PID control (Bansal et al., 2000a and 2000b; Georgiadis et al., 2002; Mohideen et al., 1996, 1997), Q parameterization method (Swartz, 2004), model predictive control (MPC) (Sakizlis, Perkins, and Piskopoulos, 2004a, 2004b), internal model control (IMC) (Chawankul, 2005), and linear quadratic Gaussian (LQG) (Meeuse and Tousain, 2002). However, they are mainly developed based on a linear model around the operating point. Thus, they will be less effective when the system has strong nonlinearity and is working in a large operating region.

2.4.3 Optimization Method

The integration problem is difficult to solve since it involves both discrete design variables and continuous control variables. To overcome this difficulty, many works have been developed. A mixed-integer dynamic optimization (MIDO) was presented to solve the integration problem that involves discrete variables, disturbances, and parametric uncertainties (Bansal et al., 2000). It was also applied to solve the integration problem with controller parameters to be tuned (Sakizlis, Perkins, and Pistikopoulos, 2004a, 2004b). The mixed integer nonlinear program (MINLP) was applied to solve the integration problem and design the heat-integrated distillation columns (Luyben and Floudas, 1994; Ross et al., 1999). An extended ant colony optimization algorithm was also proposed to solve the nonconvex or nondifferential integration problem (Schluter et al., 2009).

2.5 PROBLEMS AND RESEARCH OPPORTUNITIES

This chapter presents a brief overview on advances in robust design and its integration with control. Different methods have been studied and classified according to their fundamental nature. The underlying fundamental ideas are disclosed together with their strengths and weakness. Based on the above review, the research problems are proposed for extensive study in rest of the book.

- As discussed in the Section 2.3.1, all the existing robust designs can be classified into two categories: model-based robust designs and data-based robust designs. Since the data-based robust designs only depend on experimental data with the system knowledge fully neglected, they may have poor design performance and a high experimental cost. Moreover, they cannot handle variations of design variables. On the other hand, most of the model-based robust designs are based on the approximate first-order or second-order model, such as the Euclidean norm method, the conditional number method, and the first- and second-order moment methods. These designs may not be effective when the system has a strong nonlinearity or large uncontrollable variations, which leads to *Research Problem 1* (*Q1*). To handle this problem, new robust methods are proposed in Chapters 3 and 4 to design a nonlinear system to be insensitive to large uncontrollable variations. Moreover, there is still no solution for both model uncertainty and variations of design variables in these model-based robust designs. Thus, the robust designs cannot effectively design the partially unknown system with variations of design variables, which leads to *Research Problem 2* (*Q2*). To process Research Problem 2, robust design methods are proposed in Chapter 5 to design a system to be insensitive to both model uncertainty and variations of design variables.

- As discussed in the Section 2.3.2.1, stability design is to locate all eigenvalues of the system on the left side of the complex plane. However, the influence of parameter variations on the system eigenvalue is not considered, which leads to *Research Problem 3* (*Q3*). Moreover, no method considers the effect of model uncertainty on stability and robustness, which leads to *Research Problem 4* (*Q4*). Thus, novel robust design methods are proposed in Chapters 6, 7, and 8, respectively, to stabilize the system and to minimize variations of all eigenvalues caused by parameter variations and model uncertainty.
- As discussed in the Section 2.3, there is not much progress on design for control, namely, to have an easily controlled dynamics through process design. The robustness of the system is usually achieved through online control. However, the desired robustness of a system comes more from its inherent robust property that can only be achieved through system design. These limitations lead to *Research Problem 5* (*Q5*). Thus, a design-for-control-based integration method is proposed in Chapter 9 to obtain an easily controlled dynamics through system design, and integrate the merits of both robust design and online control for a robust pole placement under parameter uncertainty.
- As discussed in the Section 2.3, most existing approaches for the integration of design and control are only valid in the vicinity of the operation condition, since the integration is based on the linear nominal model obtained by the local approximation method. It will be difficult to obtain the accurate process model when the nonlinear process works in a large operating region. Another disadvantage is that design optimization becomes extremely difficult since the objective function in the integration is often nonconvex or nondifferential and affected by both continuous and discrete design variables. These difficulties lead to *Research Problem 6* (*Q6*). Thus, an effective modeling method will be developed in Chapter 10 to approximate the nonlinear system in a large operating region, upon which a suitable control method will be employed to handle this nonlinear system. The nonconvex or nondifferential integration problem involved will be handled by a newly proposed global optimization method.

PART II

ROBUST DESIGN FOR STATIC SYSTEMS

CHAPTER 3

VARIABLE SENSITIVITY BASED ROBUST DESIGN FOR NONLINEAR SYSTEM

For the robustness of a nonlinear system with parameter variation, variable sensitivity based deterministic and probabilistic robust design approaches are presented in this chapter. Simulation examples are used to illustrate the effectiveness of design methods.

3.1 INTRODUCTION

Deterministic robust designs are to minimize the worst case of performance under variations. Most of the deterministic approaches improve system robustness with the gradient information of variables and parameters, such as, condition number and Euclidean norm of the sensitivity matrix to measure the robustness of the system (Ting and Long, 1996; Zhu and Ting, 2001; Caro, Bennis, and Wenger, 2005), because of its simplicity and low computational cost. However, these gradient information-based robust designs are only based on the first-order or second-order approximate model developed through local linearization. This linear approximation may result in the design being less effective due to larger approximation errors, especially when the system has strong nonlinearity under slightly large uncontrollable variations. Two other common deterministic robust design approaches are sensitivity region measure method (Li, Azarm, and Boyars, 2006; Gunawan and Azarm, 2004, 2005a, 2005b) and robust space search method (Parkinson, 2000). However, these methods are complex and cost a high computation, especially for the high-dimensional nonlinear

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system. Moreover, the robust space search method needs to have known tolerances of parameters. Thus, an effective approach is needed for the deterministic robust design to handle the strongly nonlinear system to be robust to uncontrollable variations.

Probabilistic robust design approaches use probabilistic information of variables, usually their mean and variance, to improve system robustness (Al-Widyan and Angeles, 2005; Du and Chen, 2000). These approaches approximate systems with a linear or second-order model, so they can work well for the weakly nonlinear system under small random variations. However, when the system has strong nonlinearity as well as large random variations, these approaches may be less effective due to large approximation errors. To the best of our knowledge, there is little work reported for robust design of the strongly nonlinear system under large random variations. Thus, an effective approach should be developed to deal with the above problem.

In this chapter, two different robust designs, one in deterministic nature and another in probabilistic nature, are presented to improve the robustness of the nonlinear system against uncontrollable variations. Since they consider the nonlinear influence in a large design region, they can effectively improve the robustness of the nonlinear system despite uncontrollable variations. In these methods, the nonlinear system is first formulated into a linear structure, which will be easy to handle by well-developed robust design methods. This linear structure has a variable sensitivity matrix to reflect the influence of all nonlinear terms. Then, the bounds of both the variable sensitivity matrix and its singular values can be calculated in a large design region. Finally, with the variable sensitivity information incorporated, the influence of uncontrollable variations to the performance can be minimized under the framework of the traditional robust design. Both fundamental analysis and numerical simulation will demonstrate effectiveness of the presented robust design methods.

3.2 DESIGN PROBLEM FOR NONLINEAR SYSTEMS

Many systems in real world are often of strong nonlinearity with uncontrollable variations. This kind of systems can be expressed in a general format:

$$Y = f(s) \tag{3.1}$$

where $Y = [y_1 \cdots y_m]^T$ represents performance vector; $f(s) = [f_1(s) \cdots f_m(s)]^T$ is the nonlinear model; the variable $s = [d, p]^T$ includes two parts: the controllable design variable vector $d = [d_1 \cdots d_n]^T$, whose nominal value can be selected between the upper and lower bounds, and the model parameter vector $p = [p_1 \cdots p_l]^T$ with the uncontrollable variation Δp around its nominal value p_0. The variation in variable s is also uncontrollable and expressed as $\Delta s = s - s_0$ with the nominal value $s_0 = [d, p_0]$. For convenience, $f(s)$ is simply denoted as f in this chapter.

There are some difficulties in designing the robustness of the nonlinear system (Equation 3.1) under uncontrollable variation Δs due to the following reasons.

(a) influence of the nonlinearity to the performance; and
(b) interaction of variation Δs and the nonlinearity.

3.2.1 Problem in Deterministic Design

When the probabilistic information of Δs is unknown, deterministic robust designs are employed to design system robustness. Usually, common deterministic robust designs, such as Euclidean norm method or condition number method, are used to minimize the influence of uncontrollable variations to the performance through the linear model. This linear model is obtained by local linearization approach, where Taylor series expansion method is usually used, and expressed as

$$\Delta\tilde{Y} = J_0 \cdot \Delta s \tag{3.2}$$

where the sensitivity matrix $J_0 = \left.\frac{\partial f}{\partial s}\right|_{s=s_0}$ and $s_0 = [d_0, p_0]^T$ with the nominal values d_0 and p_0. It is well known that this linear approximation is only effective around the neighborhood of the design point.

Obviously, these common methods can work well for linear system or the weakly nonlinear system because the linear model (Equation 3.2) can have a reasonably well approximation, as shown in Figure (3.1a). However, when the system is strongly nonlinear, it will produce a larger approximation error Δe between the linear model (Equation 3.2) and the nonlinear system (Equation 3.1), especially when the uncontrollable variation Δs is large. The approximation error Δe can make the common method less effective because robustness is measured by the performance variation $\Delta\tilde{Y}$ of the linear model (Equation 3.2). For example, for the point A in Figure (3.1b), common robust designs measure the robustness with the performance variation $\Delta\tilde{Y}$ calculated from the linear model (Equation 3.2). However, the actual performance variation ΔY is $\Delta\tilde{Y} + \Delta e$, that is much larger than the estimated performance variation $\Delta\tilde{Y}$. Thus, the methods will be less effective if the approximation error Δe is larger. To overcome this disadvantage, an effective robust design approach should be developed to consider the robustness of the nonlinear system under larger uncontrollable variation Δs.

3.2.2 Problem in Probabilistic Design

When the probabilistic information of Δs can be gained, probabilistic robust designs are employed to design system robustness. Generally, the random variation $\Delta s = [\Delta d,$

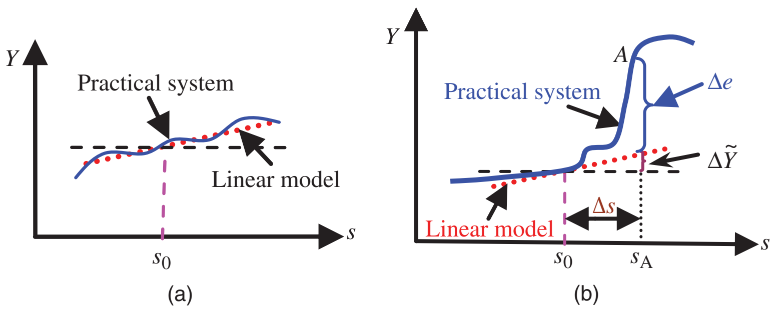


FIGURE 3.1 Deterministic robust design for nonlinear system: (a) weakly nonlinear system; (b) strongly nonlinear system

Δp] follows Gaussian distribution with a nonzero mean and a nonidentical standard deviation. The expected value μ_s and the covariance matrix σ_s of the random variation Δs can be expressed as

$$\mu_s = E[\Delta s] \tag{3.3}$$

$$\sigma_s = E[(\Delta s - \mu_s)(\Delta s - \mu_s)^T] = E[\Delta s \Delta s^T] - \mu_s \mu_s^T \tag{3.4}$$

where $E[\cdot]$ is the expected-value operator.

Usually, traditional probabilistic robust designs (Al-Widyan and Angeles, 2005) are to minimize performance covariance based on the linear model (Equation 3.2). Obviously, from Equations 3.2 and 3.3, the expected value $\tilde{\mu}_{\tilde{Y}}$ of the estimated performance $\Delta\tilde{Y}$ can be expressed as

$$\tilde{\mu}_{\tilde{Y}} = E[\Delta\tilde{Y}] = J_0 \mu_s \tag{3.5}$$

and the corresponding covariance matrix $\tilde{\sigma}_{\tilde{Y}}$ of $\Delta\tilde{Y}$ is derived as

$$\tilde{\sigma}_{\tilde{Y}} = E[(\Delta\tilde{Y} - \mu_Y)(\Delta\tilde{Y} - \mu_Y)^T] \tag{3.6}$$

Inserting Equations 3.2 and 3.5 into Equation 3.6, the covariance matrix $\tilde{\sigma}_{\tilde{Y}}$ may be rewritten as

$$\begin{aligned} \tilde{\sigma}_{\tilde{Y}} &= E\left[J_0(\Delta s - \mu_s)(\Delta s - \mu_s)^T J_0^T\right] \\ &= J_0 \cdot E[(\Delta s - \mu_p)(\Delta s - \mu_s)^T] \cdot J_0^T \end{aligned} \tag{3.7}$$

Inserting Equation 3.4 into Equation 3.7, the covariance matrix $\tilde{\sigma}_{\tilde{Y}}$ is rewritten as

$$\tilde{\sigma}_{\tilde{Y}} = J_0 \sigma_s J_0^T \tag{3.8}$$

Then, traditional probabilistic robust designs are to minimize the estimated covariance matrix $\tilde{\sigma}_{\tilde{Y}}$.

Obviously, when the system (Equation 3.1) is linear or weakly nonlinear, traditional probabilistic robust design methods can effectively work, since the model (Equation 3.2) can approximate the system (Equation 3.1) well, as indicated in Figure 3.2a). However, when the system (Equation 3.1) has strong nonlinearity and works under large random variation Δs, traditional methods will be less effective due to large approximation error between this linearization model (Equation 3.2) and the system (Equation 3.1). As an example, at point s_0 of Figure 3.2b), the traditional methods measure the system robustness with the estimated performance covariance $\tilde{\sigma}$. But the actual covariance σ could be very different to the estimated performance covariance $\tilde{\sigma}$ because of the nonlinear effect. This difference can make the traditional methods less effective for the strongly nonlinear system.

Thus, an effective probabilistic robust design approach should be developed for the nonlinear system under larger random variation.

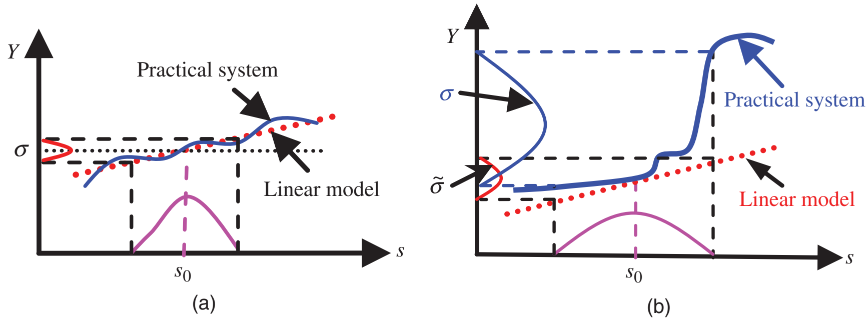


FIGURE 3.2 Probabilistic robust design for nonlinear system: (a) weakly nonlinear system; (b) strongly nonlinear system

3.3 CONCEPT OF VARIABLE SENSITIVITY

The concept and the basic idea of variable sensitivity are briefly explained below. As illustrated in Figure 3.3, given the nonlinear system (Equation 3.1), the variable sensitivity J is formulated such that $\Delta Y = \Delta f(s) = J\Delta s$ with $J \in [J_{\min}, J_{\max}]$ as $s \in [s_l, s_u]$. The performance variation ΔY of any given point in $s \in [s_l, s_u]$ may be expressed as the product of the variation Δs and the sensitivity J. This sensitivity J is equal to the gradient of the straight line between the given point and the design point. For example, the performance variation ΔY at point A relative to design point B is equal to $\Delta Y = J_{AB}(s_A - s_0)$, where J_{AB} is the sensitivity to be the same with the gradient of the line AB. Thus, all performance variations may be expressed as $\Delta Y = J\Delta s$ in $s \in [s_l, s_u]$, while the sensitivity J at different points may have a different value. Obviously, maximal and minimal values of the variable sensitivity in $s \in [s_l, s_u]$ can be figured out through the system model and denoted as $J \in [J_{\min}, J_{\max}]$. This method for processing variable sensitivity has the following advantages:

- All nonlinear terms are fully formulated into the variable sensitivity matrix without approximation.
- A linear structure is constructed, which is easy to handle by the well-developed robust design approaches.

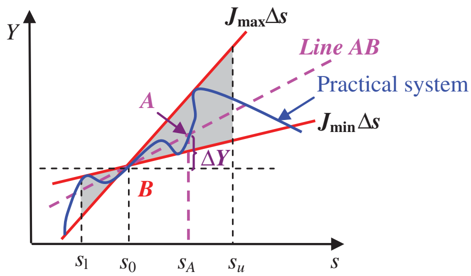


FIGURE 3.3 The method for processing variable sensitivity

3.4 VARIABLE SENSITIVITY BASED DETERMINISTIC ROBUST DESIGN

A robust design approach is briefly pictured in Figure 3.4 to design the nonlinear system to be robust to uncontrollable variation. First, the nonlinear system is modeled as a linear structure using the variable sensitivity approach, which will make the design relatively easier by using the well-developed robust design theories. Moreover, since the sensitivity matrix of this linear structure is formulated to consider the influence of the nonlinear terms, this matrix would be variable, instead of the constant matrix J_0 generated in common deterministic robust designs. Then, a variable sensitivity based robust design is developed to minimize the influence of the uncontrollable variation to the performance. Since this presented robust design method considers the effect of the nonlinearity in a larger design region, the designed system will be robust under uncontrollable variations.

3.4.1 Robust Design for Single Performance/Single Variable

Under the single performance function Y and the single variable s, the sensitivity matrix J is a scalar. The robust design for this case is relatively simple.

3.4.1.1 Variable Sensitivity Based Modeling First, the original system (Equation 3.1) can be easily formulated in the following linear structure around the design point s_0

$$\Delta Y = J \Delta s \tag{3.9}$$

with $J = \frac{f(s)-f(s_0)}{s-s_0}$.

When s is close to s_0, J is equal to J_0, as defined in (Equation 3.2) so that the presented method is the same with traditional deterministic robust design methods. However, when s has a slightly large variation around s_0, J will be variable under different variables s. It is clear that this variable sensitivity matrix J incorporates the nonlinear information of the system. Thus, this linear structure (Equation 3.9) can

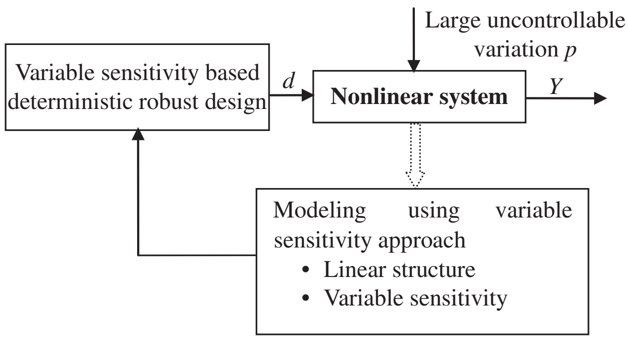


FIGURE 3.4 Variable sensitivity based robust design methodology

well express the nonlinear system in a large design region. Also, it is clear that the traditional deterministic robust design becomes a special case of the presented design method.

Then, the minimal value $J_{\min}$ and maximal value $J_{\max}$ of J can be obtained through the following optimizations when $\Delta s \in [\Delta s_l, \Delta s_u]$:

$$J_{\max} = \max_{\Delta s \in [\Delta s_l, \Delta s_u]} J, \quad J_{\min} = \min_{\Delta s \in [\Delta s_l, \Delta s_u]} J \tag{3.10}$$

Thus, J will be bounded in $[J_{\min}, J_{\max}]$.

Example 3.1: This simple example is used to show how the variable sensitivity approach works. The system is described as follows:

$$Y = f(s) = 3s^2 \quad \text{and} \quad s \in [1, 3] \tag{3.11}$$

From Equation 3.11, the performance variation ΔY at the design point $s_0 = 2$ can be derived as

$$\Delta Y = f(s) - f(s_0) = 3s^2 - 3s_0^2 = (3s + 3s_0)\Delta s \tag{3.12}$$

Thus, its sensitivity matrix J is

$$J = 3s + 3s_0 \tag{3.13}$$

and the maximal and minimal values of the sensitivity matrix J are obtained from the optimization (Equation 3.10) as

$$J_{\min} = 9 \text{ and } J_{\max} = 15 \tag{3.14}$$

Note: From Equation 3.13, it is clear that J is equal to J_0 defined in Equation 3.2 when s is close to s_0. Thus, the linear model (Equation 3.2) is a special case of the variable sensitivity model (Equation 3.9). ■

3.4.1.2 Variable Sensitivity Based Robust Design

Variable sensitivity based robust design aims to minimize the worst case of the performance variation ΔY. Two different cases are discussed as follows.

1. ***Design for the same sign of J_{max} and J_{min}***

   In this case, the worst-case $\Delta Y_{\max}$ of the performance variation ΔY is equal to $\max(|J_{\min}|, |J_{\max}|) \cdot \Delta s$, as shown in Figure 3.5, where $\max(|J_{\min}|, |J_{\max}|)$ is actually the maximal value between $|J_{\min}|$ and $|J_{\max}|$. For example, if $J_{\max}$ and $J_{\min}$ are minus, then $\max(|J_{\min}|, |J_{\max}|)$ is $|J_{\min}|$. Otherwise,

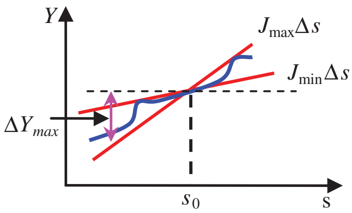


FIGURE 3.5 Robust design for the same sign of $J_{\max}$ and $J_{\min}$

$\max(|J_{\min}|, |J_{\max}|)$ is $|J_{\max}|$. Thus the robust design is equivalent to the minimization of $\max(|J_{\min}|, |J_{\max}|)$ without controlling Δs.

$$\min_{d_0} \max(|J_{\min}|, |J_{\max}|)$$
$$st.\ h(s) = 0, l(s) \leq 0 \tag{3.15}$$

where $h(s)$ and $l(s)$ are constraints from other design aspects. Its solution can make the nonlinear system robust to uncontrollable variations.

2. ***Design for the different sign of J_{max} and J_{min}***
   Similarly, the worst-case $\Delta Y_{\max}$ is equal to $|J_{\max} - J_{\min}| \cdot \Delta s$, as shown in Figure 3.6. Thus the robust design problem is equivalent to the minimization of $|J_{\max} - J_{\min}|$

$$\min_{d_0} |J_{\max} - J_{\min}|$$
$$st.\ h(s) = 0, l(s) \leq 0 \tag{3.16}$$

Its solution can make the nonlinear system robust to uncontrollable variations.

3.4.2 Robust Design for Multiperformances/Multivariables

When there are multiple performance functions Y and multiple variables s, the performance variations ΔY can be easily expressed as below:

$$\Delta Y = \Delta f(s) = J\Delta s \tag{3.17}$$

where the variable sensitivity matrix J becomes an $m \times (n + l)$ matrix.

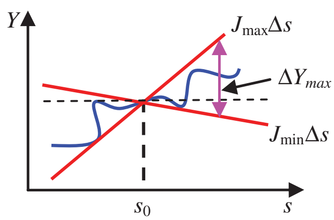


FIGURE 3.6 Robust design for the different sign of $J_{\max}$ and $J_{\min}$

Let $\Delta s \in [\Delta s_l, \Delta s_u]$ and then the system model may be used to calculate the minimal value $J_{i,j}^{\min}$ and the maximal value $J_{i,j}^{\max}$ of $J_{i,j}$, which is an element of J:

$$J_{i,j}^{\max} = \max_{\Delta s \in [\Delta s_l, \Delta s_u]} J_{i,j},\ J_{i,j}^{\min} = \min_{\Delta s \in [\Delta s_l, \Delta s_u]} J_{i,j} \tag{3.18}$$

Then, the performance variations ΔY in Equation 3.17 may be rewritten as

$$(\Delta f(s))^T \Delta f(s) = (\Delta s)^T B \Delta s \tag{3.19}$$

with $B = J^T J$.

From Equations 3.18 and 3.19, matrix B is a bound matrix with its bound elements derived as

$$B_{i,j} \in \left[B_{i,j}^{\min}, B_{i,j}^{\max}\right] \tag{3.20}$$

Moreover, according to the singular value decomposition (SVD) theory, any real symmetric matrix B may be decomposed as

$$B = \zeta diag(\delta_1, \ldots, \delta_{n+l})\zeta^T \tag{3.21}$$

where δ_i is the singular value of J, and the corresponding orthogonal eigenvector is denoted as ζ_i, which is one element of the vector $\zeta = [\,\zeta_1 \cdots \zeta_{n+l}\,]$.

Since matrix B varies within its bounds, its singular values are also bounded as

$$\delta_i \in \left[\delta_i^{\min},\ \delta_i^{\max}\right] \tag{3.22}$$

This bound can be calculated from matrix B.

Example 3.2: The nonlinear system is described as

$$Y = f(s) = \begin{bmatrix} s_1^2 + s_2^3 \\ 3s_1 + 2s_2^2 \end{bmatrix} \text{ and } s = [s_1, s_2]^T \text{ with } s_1 \in [1, 3], s_2 \in [1, 3] \tag{3.23}$$

From Equation 3.23, the performance variation ΔY around the design point $s_0 = [2, 2]^T$ can be formulated as

$$\begin{aligned} \Delta Y &= f(s) - f(s_0) \\ &= \begin{bmatrix} s_1^2 + s_2^3 - s_{1,0}^2 - s_{2,0}^3 \\ 3s_1 + 2s_2^2 - 3s_{1,0} - 2s_{2,0}^2 \end{bmatrix} \\ &= \begin{bmatrix} s_1 + s_{1,0} & s_2^2 + s_{2,0}s_2 + s_{2,0}^2 \\ 3 & 2(s_2 + s_{2,0}) \end{bmatrix} \begin{bmatrix} \Delta s_1 \\ \Delta s_2 \end{bmatrix} \end{aligned} \tag{3.24}$$

Thus, its sensitivity matrix is

$$J = \begin{bmatrix} s_1 + s_{1,0} & s_2^2 + s_{2,0}s_2 + s_{2,0}^2 \\ 3 & 2(s_2 + s_{2,0}) \end{bmatrix} \tag{3.25}$$

Thus, the bound of matrix J can be calculated as below

$$J_{1,1} \in [3, 5], J_{1,2} \in [7, 19], J_{2,2} \in [6, 10] \tag{3.26}$$

Then, matrix B may be figured out

$$B = \begin{bmatrix} J_{1,1}^2 + J_{2,1}^2 & J_{1,1}J_{1,2} + J_{2,1}J_{2,2} \\ J_{1,1}J_{1,2} + J_{2,1}J_{2,2} & J_{1,2}^2 + J_{2,2}^2 \end{bmatrix} \tag{3.27}$$

Thus, the bound of matrix B can be calculated as below

$$B_{1,1} \in [18, 34], B_{1,2} = B_{2,1} \in [39, 125], B_{2,2} \in [85, 461] \tag{3.28}$$

Moreover, the singular value of matrix J can be expressed as

$$\delta_1 = 0.5(B_{2,2} + B_{1,1} + \sqrt{(B_{2,2} + B_{1,1})^2 - 4\left(B_{2,2}B_{1,1} - B_{1,2}^2\right)} \tag{3.29a}$$

$$\delta_2 = 0.5(B_{2,2} + B_{1,1} - \sqrt{(B_{2,2} + B_{1,1})^2 - 4\left(B_{2,2}B_{1,1} - B_{1,2}^2\right)} \tag{3.29b}$$

Thus, the bound of the singular values can be calculated as below

$$\delta_1 \in [97, 490.6], \quad \delta_2 \in [3.96, 7.72] \tag{3.30}$$

Note: From Equation 3.25, it is clear that J is equal to J_0 defined in Equation 3.2 when s is close to s_0. Thus, the linear model (Equation 3.2) is a special case of the variable sensitivity model (Equation 3.17) even for the system with multiperformances/multivariables. ■

Moreover, inserting Equation 3.21 into Equation 3.19, the performance Y_r may be expressed as follows

$$Y_r = \sum_{i=1}^{n+l} \delta_i (z_i)^2 \tag{3.31}$$

with $Y_r = \|\Delta Y\|_2^2 = (\Delta f(s))^T \Delta f(s)$ and $[z_1, \ldots, z_{n+l}]^T = \zeta^T \Delta s$.

Since both the singular values and the eigenvectors in Equation 3.31 vary within a bound, the performance Y_r in the m-dimensional space is a set of hyper-ellipsoids as defined in Equation 3.31. Its two-dimensional projection is depicted in Figure 3.7.

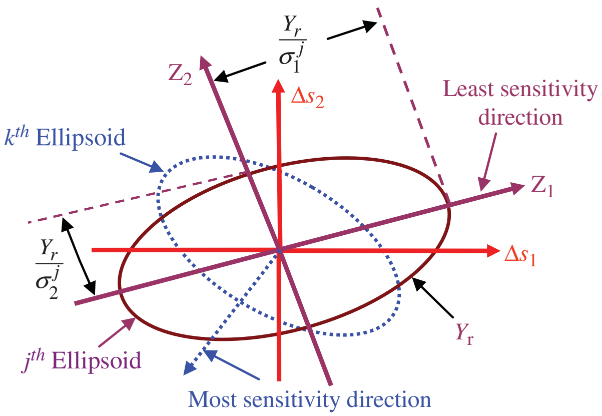


FIGURE 3.7 Multiple sensitivity ellipsoids

Every hyper-ellipsoid has the following characteristics:

(a) The performance Y_r defined in (Equation 3.31) is the same for every point on the hyper-ellipsoid.
(b) The length of the ith principal axis of the jth hyper-ellipsoid is Y_r/δ_i^j, where the singular value δ_i^j is a value bounded in $[\delta_i^{\min}, \delta_i^{\max}]$. The smaller δ_i^j is, the longer the ith principal axis will be. The longest/shortest principal axis corresponds to the least/most sensitive direction for the given hyper-ellipsoid.

For all hyper-ellipsoids,

(a) The length of the ith principal axis of all hyper-ellipsoids is bounded in $\left[\frac{Y_r}{\delta_i^{\max}}, \frac{Y_r}{\delta_i^{\min}}\right]$.
(b) The longest/shortest principal axis in all hyper-ellipsoids indicates the least/most sensitive direction for the nonlinear system.

According to the robust design theory, all principal axes in all hyper-ellipsoids should be made as long as possible, especially the shortest principal axis in all hyper-ellipsoids. The shortest principal axis in all hyper-ellipsoids corresponds to the maximal singular value $\delta_{\max}$ in all singular values. If the maximal singular value $\delta_{\max}$ can be minimized, then the shortest principal axis will achieve the largest length. So the design variable d for the robust performance can be figured out through the following min-max optimization.

$$\begin{gathered}\min_{d_0} \max_i \left(\delta_i^{\max}\right) \\ st.\ h(s) = 0,\ l(s) \leq 0\end{gathered} \tag{3.32}$$

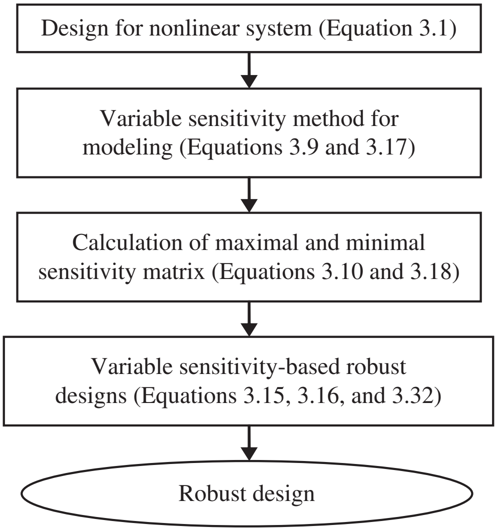


FIGURE 3.8 Deterministic robust design flowchart

The solution of the optimization (Equation 3.32) can achieve the robustness under uncontrollable variations.

3.4.3 Design Procedure

The presented deterministic robust design methodology is summarized in Figure 3.8. Nonlinear system is first formulated as a linear structure just as Equations 3.9 and 3.17, using the variable sensitivity method. Since its sensitivity matrix incorporates the effect of the nonlinear terms, it can express the nonlinear system better than the common linearization approach with a constant sensitivity matrix J_0. Then, the bound of this sensitivity matrix is calculated from the system model. For multiperformances/multivariables system, the maximal and minimal singular values should also be calculated as Equation 3.22. Finally, based on the obtained sensitivity matrix, the variable sensitivity based robust approaches as given in Equations 3.15, 3.16, and 3.32 can achieve the robustness of the nonlinear system under uncontrollable variations.

3.5 VARIABLE SENSITIVITY BASED PROBABILISTIC ROBUST DESIGN

A novel probabilistic robust design approach is proposed in Figure 3.9 to design the robustness of the nonlinear system under large random variations. Initially, the nonlinear system is formulated under a linear structure using the variable sensitivity approach. Then, the nonlinear system will be designed for robustness through minimization of performance covariance. Since this proposed approach considers the

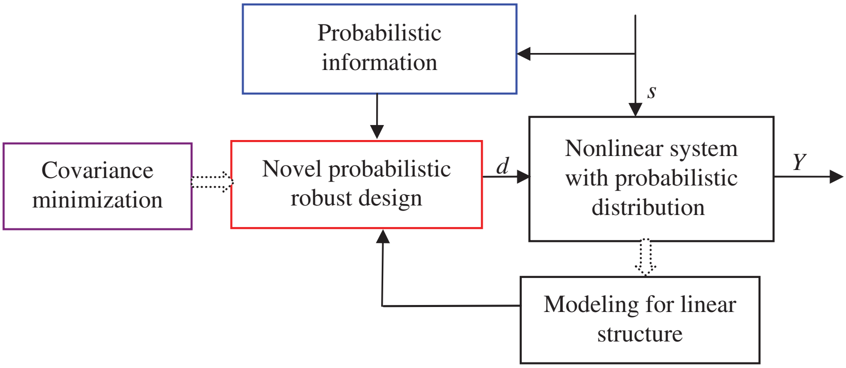


FIGURE 3.9 New probabilistic robust design method

effect of the nonlinearity and the probabilistic information of variations, it can ensure the robustness of the nonlinear system even if large random variation exists.

3.5.1 Single Performance Function Under Single Variables

First, the variable sensitivity based modeling method is used to obtain its model (Equation 3.9). Then, an effective approach is developed to design the nonlinear system to be robust. According to the standard definition, the expected value μ_Y and the covariance matrix σ_Y can be expressed as

$$\mu_Y = J\mu_s, \quad \sigma_Y = J^2\sigma_s \tag{3.33}$$

The robust design requires the covariance matrix σ_Y to be minimized. Since the term σ_s on the right of Equation 3.33 cannot be controlled by the designer, this robust design becomes the following optimization problem

$$\min_d J^2 \tag{3.34}$$

Since J is bounded in $[J_{\min}, J_{\max}]$ according to Equation 3.10, the robust design can be transformed into

$$\begin{aligned} &\min_d \max(|J_{\max}|^2,\ |J_{\min}|^2) \\ &s.t. \quad h(s) = 0, \quad l(s) \le 0 \end{aligned} \tag{3.35}$$

where $\max(|J_{\max}|^2,\ |J_{\min}|^2)$ is to select a maximal value between $|J_{\max}|^2$ and $|J_{\min}|^2$. This min-max optimization is a most common problem in robust design and its objective is to minimize the worst case of performance variation. When this worst case is minimized, all performance variations will be minimal and thus guarantee the system to be robust.

3.5.2 Single Performance Function Under Multivariables

In case of the single performance function Y and multivariables s, the variable sensitivity matrix J in Equation 3.9 becomes a $1 \times (n+l)$ vector. Then, the minimal and maximal values of the ith element of J can be calculated from the variable sensitivity model as follows

$$J_{i,\max} = \max_{\Delta s \in [\Delta s_l, \Delta s_u]} J_i, \quad J_{i,\min} = \min_{\Delta s \in [\Delta s_l, \Delta s_u]} J_i \tag{3.36}$$

The expected value μ_Y and the covariance matrix σ_Y are expressed as

$$\mu_Y = J\,\mu_s, \quad \sigma_Y = J\sigma_s J^T \tag{3.37}$$

According to the matrix theory, we have

$$\sigma_Y = J\sigma_s J^T \equiv \|\tilde{J}\|^2_{\sigma_s} \tag{3.38}$$

where $\tilde{J}$ denotes the single row of J in vector form, that is, as a column array, and $\|\cdot\|_{\sigma_s}$ is the weighted Euclidean norm of vector $(\cdot)$ with respect to the positive-define matrix σ_s. Through the matrix transformation, the following inequality is satisfied

$$\|\tilde{J}\|^2_{\sigma_s} = \tilde{J}^T\sigma_s\tilde{J} = tr(\tilde{J}\tilde{J}^T\sigma_s) \le tr(\tilde{J}\tilde{J}^T)tr(\sigma_s) = \|\tilde{J}\|^2_2\, tr(\sigma_s) = JJ^T tr(\sigma_s) \tag{3.39}$$

where $||\cdot||_2$ and $tr(\cdot)$ are the Euclidean norm and the trace of $(\cdot)$ respectively.

Thus, from Equations 3.38 and 3.39, the covariance matrix σ_Y is bounded as

$$\sigma_Y \le JJ^T tr(\sigma_s) \tag{3.40}$$

Obviously, the designer cannot control $tr(\sigma_s)$, so this robust design actually becomes the minimization of JJ^T. From Equation 3.36, $J^T J$ will be constrained by

$$JJ^T \le \sum_{i=1}^{n} \max(|J_{i,\max}|^2,\ |J_{i,\min}|^2) \tag{3.41}$$

Thus, the minimization of $J^T J$ can be transformed into the following optimization,

$$\begin{aligned} &\min_d \sum_{i=1}^{n} \max(|J_{i,\max}|^2,\ |J_{i,\min}|^2) \\ &s.t. \quad h(s) = 0, \quad l(s) \le 0 \end{aligned} \tag{3.42}$$

The solution of Equation 3.42 can ensure the robustness of the nonlinear system even if large random variation exists.

3.5.3 Multiperformance Functions Under Multivariables

In case of multiperformance functions Y and multivariables s, the variable sensitivity matrix J in Equation 3.9 becomes an $m \times (n + l)$ matrix. Let $\Delta s \in [\Delta s_l, \Delta s_u]$ and then the minimal value $J_{i,j}^{\min}$ and the maximal value $J_{i,j}^{\max}$ of the element $J_{i,j}$ of J can be calculated as

$$J_{i,j}^{\max} = \max_{\Delta s \in [\Delta s_l, \Delta s_u]} J_{i,j}, \quad J_{i,j}^{\min} = \min_{\Delta s \in [\Delta s_l, \Delta s_u]} J_{i,j} \tag{3.43}$$

The expected value μ_Y and the covariance matrix σ_Y of the performance variation ΔY can be expressed as

$$\mu_Y = J\mu_s, \quad \sigma_Y = J\sigma_s J^T \tag{3.44}$$

According to the matrix theory, the covariance matrix σ_Y may be rewritten as

$$\|\sigma_Y\|_F \leq \sqrt{\frac{1}{m} tr(J^T J) tr(\sigma_s)} \tag{3.45}$$

where $\|\cdot\|_F$ is the Frobenius norm of $(\cdot)$. Thus, minimization of the performance covariance can be transformed into minimization of its upper bound in Equation 3.45. Since the term $tr(\sigma_s)\sqrt{\frac{1}{m}}$ cannot be controlled by the designer, this robust design changes to the following optimization

$$\min_d tr(J^T J) \tag{3.46}$$

According to the matrix norm theory, $tr(J^T J)$ may be rewritten as

$$tr(J^T J) = \|J\|_F^2 = \sum_{i=1}^{n+l} \delta_i^2 \tag{3.47}$$

where δ_i is the singular value of J.

According to the discussion in Section 3.3.2 and Equation 3.22, its singular values are bounded as $\delta_i \in [\delta_i^{\min},\ \delta_i^{\max}]$. Thus, the robust design may be figured out through the following optimization,

$$\begin{aligned} &\min_{d_0} \max_{\delta_i \in [\delta_i^{\min},\ \delta_i^{\max}]} \left(\sum_{i=1}^{n+l} \delta_i^2 \right) \\ &s.t. \quad h(s) = 0, \quad l(s) \leq 0 \end{aligned} \tag{3.48}$$

The solution of the optimization problem (Equation 3.48) can ensure the robustness of the nonlinear system even if there is large random variation.

Note: The min-max optimization appeared in the proposed methods is a common problem in the system design/control. When their objective functions are convex function, they can easily be solved by the traditional gradient algorithm. When they are nonconvex and nonlinear functions, some intelligent algorithms or nontraditional methods, such as particle swarm optimization (PSO) and genetic algorithm (GA), could be used to explore the near-optimal solution.

3.6 CASE STUDY

3.6.1 Deterministic Design Cases

The presented deterministic robust design method will be compared with the common Euclidean norm method in two different cases. Since deviation from its objective can be estimated as $\|Y(d_0 + \Delta d, p_0 + \Delta d) - Y(d_0, p_0)\|_2^2$, the performance index E_r will be defined as

$$\begin{aligned} E_r = & \left\|Y(d_{0,T} + \Delta d, p_0 + \Delta p) - Y(d_{0,T}, p_0)\right\|_2^2 \\ & - \left\|Y(d_{0,p} + \Delta d, p_0 + \Delta p) - Y(d_{0,p}, p_0)\right\|_2^2 \end{aligned} \tag{3.49}$$

where $d_{0,T}$ and $d_{0,p}$ are the design variables gained by the Euclidean norm method and the presented robust design method, respectively. If the percentage of $E_r > 0$ is larger than 50%, when Δp and Δd are randomly sampled from their variation space, then the presented method is more robust than the Euclidean norm method. Otherwise, the Euclidean norm method is better.

Example 3.3: Robust design of a belt Belts are used in transmission of power between shafts with either parallel or skewed axes. The power transmitted by a belt is

$$W = f(s) \tag{3.50}$$

with $f(s) = (1 - e^{-\zeta\theta})(T - MV^2)V$.
where M and θ are the mass of the belt per unit length and the contact angle respectively, V and T are the belt speed and the tension in the belt. respectively, and ζ and W are the coefficient of friction and the transmitted power, respectively. The coefficient of friction ζ, the tension T, the nominal mass M, and the contact angle θ are 0.2, 15 Nm, 1 kg, and $\pi/4$, respectively. The design variable and the performance function are

$$s = V, \quad Y = W$$

The design task is to achieve a robust performance against the variation $\Delta s = \Delta V$ through identifying the design variable V_0 from the design space $[0.5, 2.5]$.

First, the robust design calculated by the Euclidean norm approach is

$$V_{\mathrm{T}} = 2.5 \tag{3.51}$$

Then, the presented robust design is used to design the belt under $\Delta V \in [-R, R]$. From Equation 3.31, the performance variation ΔW can be expressed as

$$\Delta W = J\Delta V = J\Delta s \tag{3.52}$$

with $J = (1 - e^{-\zeta\theta})[T - M(V^2 + VV_0 + V_0^2)]$.

The maximal value $J_{\max}$ and minimal value $J_{\min}$ of J are easily calculated as

$$\begin{aligned} J_{\min} &= (1 - e^{-\zeta\theta}) \left[T - M\left((V_0 + R)^2 + (V_0 + R)V_0 + V_0^2\right)\right], \\ J_{\max} &= (1 - e^{-\zeta\theta}) \left[T - M\left((V_0 - R)^2 + (V_0 - R)V_0 + V_0^2\right)\right] \end{aligned} \tag{3.53}$$

If $J_{\max}$ and $J_{\min}$ have the same sign, then the robust design can be figured out from Equation 3.15 under the given bound R. Otherwise, it will be figured out from Equation 3.16 under the given bound R.

For performance comparison and verification, let ΔV be an uniformly distributed random variation in $(-R, R)$ and a total of 1000 samples are taken to compare the worst case of the performance variation $(\Delta W)^2$ with respect to the variation ΔV. Designs under different variation bounds R are shown in Figure 3.10, where it is clear that the worst case of the performance variation obtained by the presented robust design is better (smaller) than the common Euclidean norm approach. Thus, the presented robust design has a better robust performance than the common Euclidean

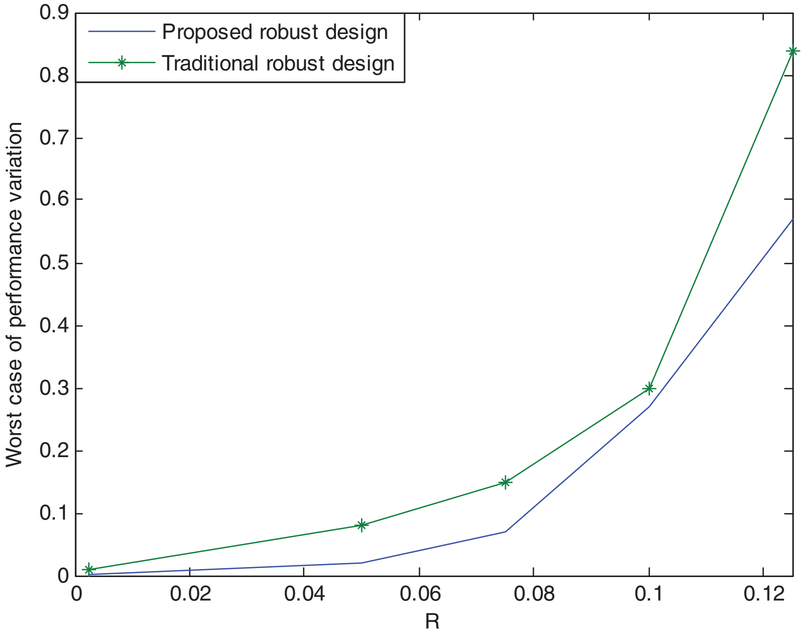


FIGURE 3.10 Comparison under different variation bounds

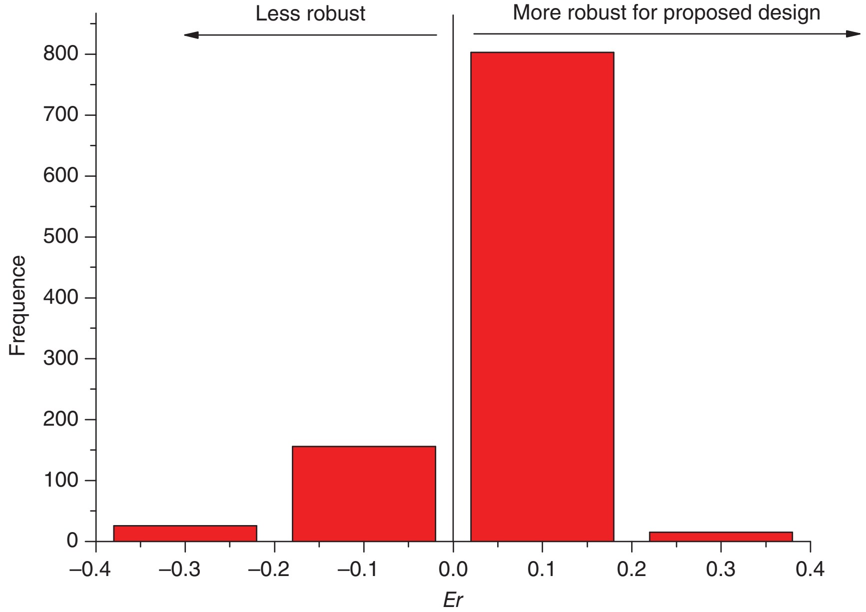


FIGURE 3.11 Performance *Er*

norm approach. This is because it considers the influence from the nonlinear term but the Euclidean norm approach does not.

Finally, the performance E_r is compared. Let $R = 0.1$ and thus ΔV be bounded in $(-0.1, 0.1)$. Under this bound, the presented robust design is $V_p = 2.24$. A total of 1000 samples are taken to compare the performance E_r. From Figure 3.11, it is clear that it has about 81.8% (for $E_r > 0$) chances to have a better design than the common one. Thus, the presented approach is more robust than the Euclidean norm method. ■

Example 3.4: Robust design of a damper The design problem of a damper is shown in Example 1.1 in Chapter 1. The design variable d, the model parameter p, and the performance functions Y are

$$d = M,\ p = C_d,\ s = [M \quad C_d]^T,\ Y = \begin{bmatrix} X \\ \phi \end{bmatrix}$$

F and w are set as 200 N and 31.4 rad/s, respectively. The model parameter C_d is equal to $C_{d_0} \pm H$ with the nominal value $C_{d_0} = 50$, and the design variable M has the variation $\Delta M \in [-R, R]$ around its design point M_0. The objective is to select the suitable design variable M_0 from $M_0 \in [2kg, 3kg]$ to make the system robust to the uncontrollable variation $\Delta s \in [\Delta C_d, \Delta M]^T$.

For the design problem (Equation 1.1) in Chapter 1, the robust design calculated by the Euclidean norm method is $M_{\mathrm{T}} = 2.5$. The presented robust design for the damper design can be easily figured out from Equation 3.32, with the given bounds H and R.

For performance comparison and verification, let ΔM and ΔC_d be uniformly distributed random variations in $(-R, R)$ and $(-H, H)$ respectively, and a total of 1000 samples are taken to compare the worst case of the performance variation $\Delta W = [(\Delta X)^2 + (\Delta\phi)^2]$. Define

$$\text{Difference of worst case} = \max(\Delta W_T) - \max(\Delta W_p) \tag{3.54}$$

where ΔW_T and ΔW_p are worst cases of the performance variation $\Delta W = [(\Delta X)^2 + (\Delta\phi)^2]$ obtained by the Euclidean norm method and the presented robust design method, respectively. Only if this difference is positive, the presented robust design has a better robust performance than the Euclidean norm method. This difference under the variation bounds R and H is shown in Figure 3.12, where the positive difference clearly indicates that the presented robust design has a better robust performance than the Euclidean norm approach.

For comparison of E_r, let $R = 0.3$ and $H = 15$ and thus ΔM and ΔC_d are bounded in $(-0.3, 0.3)$ and $(-15, 15)$ respectively. Under these bounds, the presented robust design is $M_p = 2.28$. A total of 1000 samples are taken for the comparison of performance E_r as shown in Figure 3.13, where it shows about 64.1% (for $E_r > 0$) chances to have a better design than the Euclidean norm method. Since the percentage is larger than 50%, the presented method is more robust than the Euclidean norm method.

In conclusion, the presented robust design method is more effective for the nonlinear system under uncontrollable variations. ■

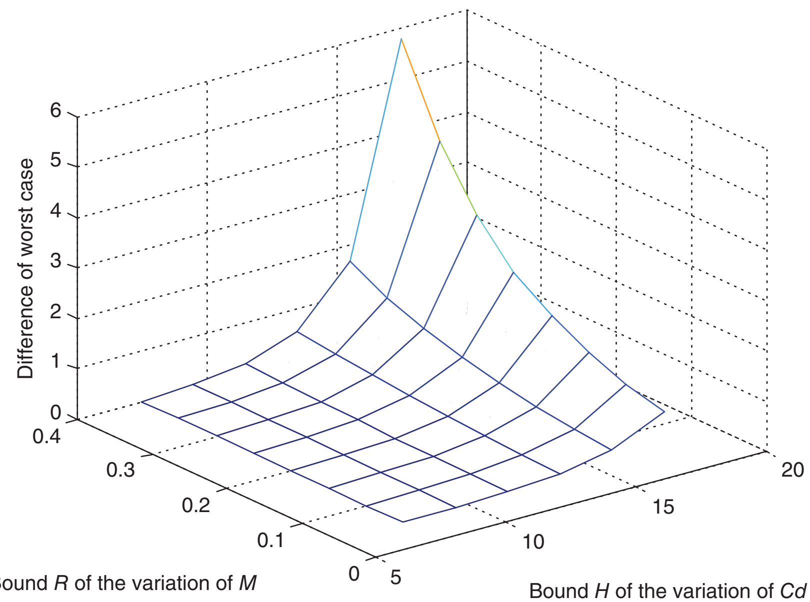


FIGURE 3.12 Difference of the worst case under variation bounds

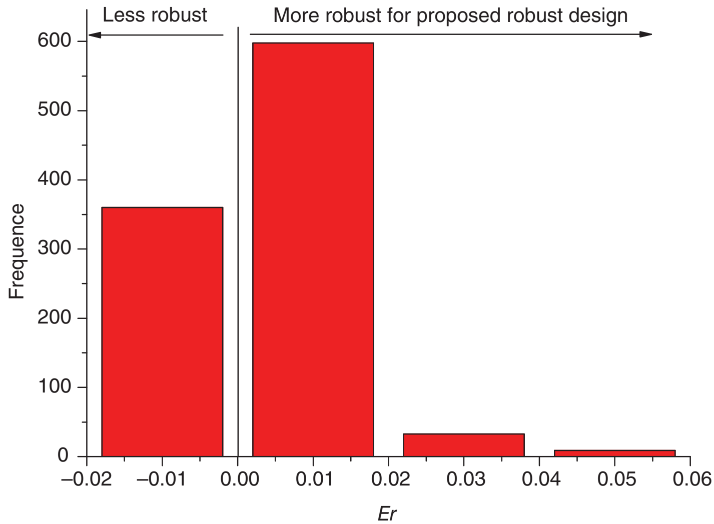


FIGURE 3.13 Performance *Er*

3.6.2 Probabilistic Design Case

The low-pass filter design example in the reference (Al-Widyan and Angeles, 2005) as shown in Figure 3.14 is taken to verify the effectiveness of the proposed probabilistic design. The design variables are the resistance R and the inductance L. The current $i(t)$ is harmonic of the form $i(t) = I\cos(wt + \phi)$, with I and ϕ as the amplitude and the phase of $i(t)$. The amplitude I is kept at a nominal value $I_0 = 10$A. V and w are the amplitude and the frequency of the excitation voltage $v(t) = V\cos wt$, respectively.

The performance Y is expressed as

$$Y = f(d) = \begin{bmatrix} \dfrac{V_0}{\sqrt{R^2 + w_0^2 L^2}} \\ \tan^{-1}\left(\dfrac{w_0 L}{R}\right) \end{bmatrix} \tag{3.55}$$

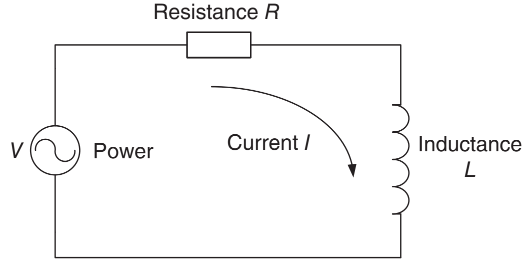


FIGURE 3.14 A low-pass filter

where the design variable d and the performance function Y are

$$d = \begin{bmatrix} R \\ L \end{bmatrix}, \quad Y = \begin{bmatrix} I \\ \phi \end{bmatrix}$$

The design variable d has variation around its nominal value and this variation follows Gaussian distribution with μ_d and σ_d. Moreover, the known nominal value V_0 and w_0 are 110 V and 60 Hz, respectively.

Usually, these random variations can be expressed as

$$\Delta R = T \times \eta \text{ and } \Delta L = H \times \eta \tag{3.56}$$

with $\eta \sim N(-0.5, 1)$, known parameters T and H.

The robust design variable d figured out by the traditional design approach (Al-Widyan and Angeles, 2005) is ($R_0 = 0.068\ \Omega$, $L_0 = 10.21$ H). The proposed robust design can be easily figured out from Equation 3.48 with the given T and H.

Then, the performance comparison can be conducted. Define a performance index,

$$E_{cov} = \|\sigma_{Y,T}\|_F - \|\sigma_{Y,p}\|_F \tag{3.57}$$

where $\sigma_{Y,T}$ and $\sigma_{Y,p}$ are the covariance σ_Y obtained by the traditional design and the proposed design, respectively. From Equation 3.57, it is clear that, only when those with E_{cov} are larger than zero, the proposed approach has a better robust performance than the traditional approach. The performance index E_{cov} under different T and H is shown in Figure 3.15, where it is clearly shown that all E_{cov} are bigger than zero.

Moreover, statistical test is carried out. Samples from the proposed robust design and the traditional robust design are averagely divided into K groups. Then, the F-norm of the covariance of the ith group is calculated as $\|\sigma_{Y,_i}\|_F$ ($i = 1, \ldots, K$). Hypothesis is set up as follows:

$$H_0 : \frac{1}{K}\sum_{i=1}^{K} \|\sigma^T_{Y,_i}\|_F = \frac{1}{K}\sum_{i=1}^{K} \|\sigma^p_{Y,_i}\|_F \text{ and } H_1\!: \frac{1}{K}\sum_{i=1}^{K} \|\sigma^T_{Y,_i}\|_F > \frac{1}{K}\sum_{i=1}^{K} \|\sigma^p_{Y,_i}\|_F \tag{3.58}$$

where $\|\sigma^T_{Y,_i}\|_F$ and $\|\sigma^p_{Y,_i}\|_F$ are the F-norm of the covariance of the ith group obtained by the traditional robust design and the proposed robust design, respectively.

The following two-sampling t-test is used to conduct the test

$$t = \frac{\frac{1}{K}\sum_{i=1}^{K} \|\sigma^T_{Y,_i}\|_F - \frac{1}{K}\sum_{i=1}^{K} \|\sigma^p_{Y,_i}\|_F}{\sqrt{(S_T^2 + S_p^2)/K}} \tag{3.59}$$

where S_T^2 and S_p^2 are the standard deviation of the covariance's F-norm of all groups obtained by the traditional method and the proposed method, respectively.

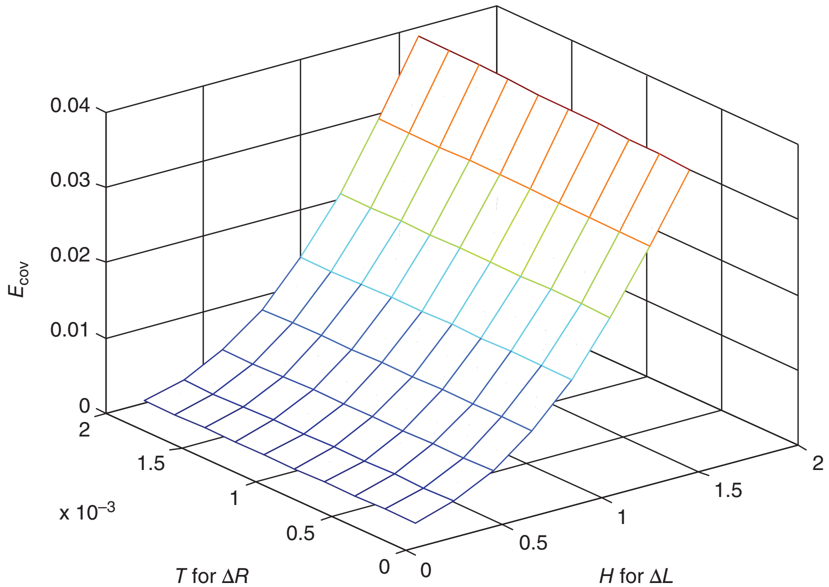


FIGURE 3.15 The performance E_{cov} under T and H

If H_0 is rejected and H_1 is accepted, the proposed method has a smaller covariance than the traditional method. Using ttest2 function of Matlab software, all H_0 are rejected and all H_1 are accepted when $T \in [0.0002, 0.002]$ and $H \in [0.2, 1.6]$. For example, since $t = 14.6$ obtained at $T = 0.0004$ and $H = 0.4$ is larger than $t_{a=0.05} = 1.81$, H_0 is rejected and H_1 is accepted. Thus, the covariance gained by the proposed method is smaller than the traditional method. This means that, from the statistical viewpoint, the proposed method will have better robust performance than the traditional method.

Finally, define

$$E_{mean} = Mean\left(\|\Delta Y_T\|_2\right) - Mean\left(\|\Delta Y_p\|_2\right) \tag{3.60a}$$

$$E_{var} = Var(\|\Delta Y_T\|_2) - Var(\|\Delta Y_p\|_2) \tag{3.60b}$$

where $Mean(\cdot)$ and $Var(\cdot)$ represent mean and variance of $(\cdot)$, and ΔY_p and ΔY_T are the performance variations obtained by the traditional design and the proposed design, respectively.

Only if both E_{mean} and E_{var} are bigger than zero, the proposed design has a better robust performance than the traditional design. E_{mean} and E_{var} under different T and H are shown in Figure 3.16, where it is clearly shown that all E_{mean} and E_{var} are bigger than zero.

Thus, from the statistical test and Figures 3.15 and 3.16, the proposed method has better robust performance than the traditional approach, because it considers the nonlinear influence and the large random variation that the traditional method does not consider.

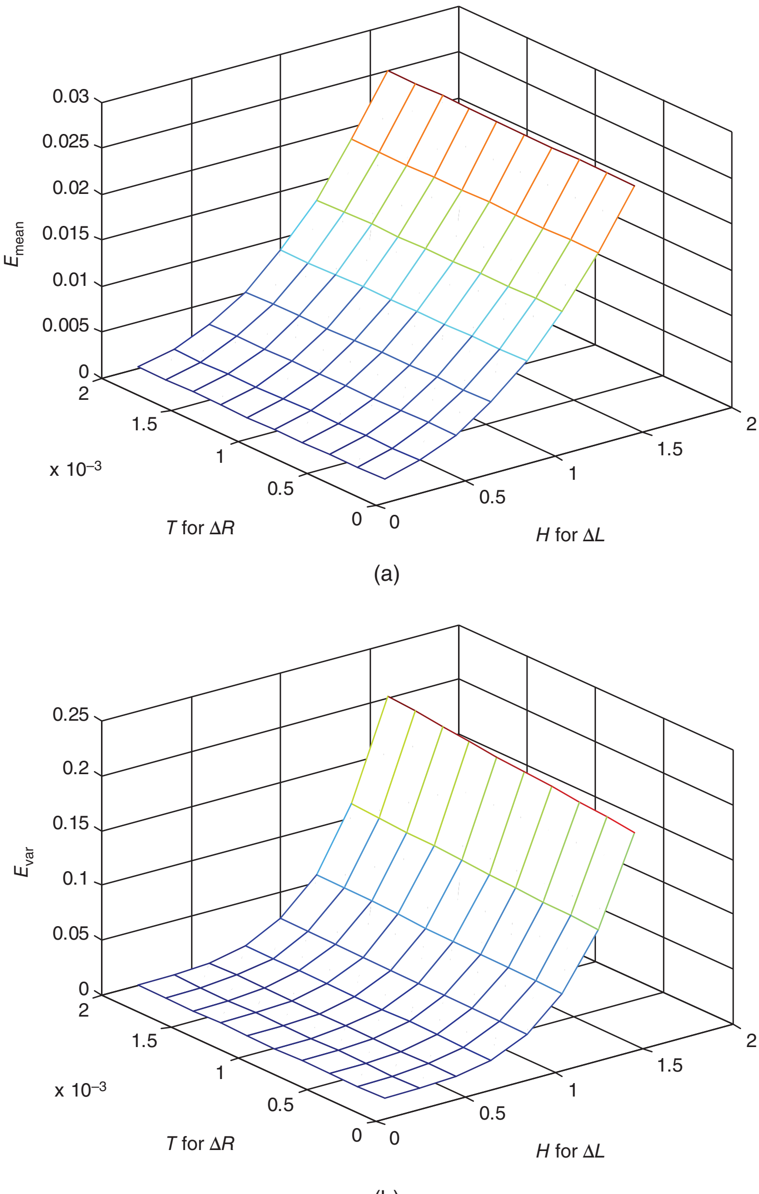


FIGURE 3.16 Comparison of the performance variation under T and H. (a) E_{mean}; (b) E_{var}

3.7 SUMMARY

In this chapter, two new approaches are presented for robust design of the nonlinear system under uncertainties. These design approaches consider not only the inherent nonlinearity of the system but also uncontrollable variations caused externally. First, the variable sensitivity approach is constructed to express the nonlinear system since it can handle nonlinear influence. Then, the design developed from the variable sensitivity, either in deterministic or probabilistic nature, can demonstrate a better robust performance even for the strongly nonlinear system under uncontrollable variations. The performance comparisons on simulation examples confirm the effectiveness of the proposed design.

CHAPTER 4

MULTI-DOMAIN MODELING-BASED ROBUST DESIGN

This chapter will develop an approach to design a strongly nonlinear system to be robust against large parameter variation. Based on the idea of linearization, complex nonlinear system in the whole parameter space can be decomposed into a series of linear systems at each subdomain. Modeling of each linear system at the subdomain would be much easier than modeling of the original system in the whole parameter space. Design of the nonlinear system under large parameter variation can be achieved through multi-optimizations over these subdomains.

4.1 INTRODUCTION

For a high quality production, mechanical systems used in complex manufacturing environment are required to have a consistent performance over a large working region. These systems may have strongly nonlinear characteristics and have to work under larger uncontrollable variations, as indicated in Figure 4.1. For simplicity, the system can be described by:

$$y = f(d, p) \tag{4.1}$$

where y represents performance vector, $f(d, p)$ is the nonlinear model, d is the design variable vector that needs to be figured out from its working pace S_d, and p is the model

System Design and Control Integration for Advanced Manufacturing, First Edition. Han-Xiong Li and XinJiang Lu.

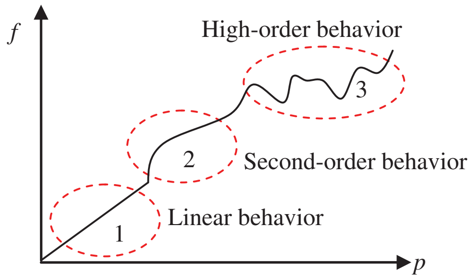


FIGURE 4.1 Model behavior at different operating domains of the parameter

parameter vector suffered from large uncontrollable variation Δp with its nominal value p_0 within its working space S_p. Obviously, this kind of system is difficult to obtain consistent performance under large variations.

Although the sensitivity-based robust design method is simple and effective for many linear or weakly nonlinear systems, they are still difficult to gain consistent performance for this strongly nonlinear system with large parameter variation. The main reasons include the following:

- Larger parameter variation is not considered in the traditional design.
- The influence of nonlinear term is not considered in most of the methods when using a linear mode to design the system, such as Euclidean norm method, conditional number method, and first- and second-order moment methods. As indicated in Figure 4.2a, this kind of method cannot produce a good design due to large model errors generated.
- Even if some methods consider the influence of nonlinear term in a small design domain, such as the robust design approach in Chapter 3, the result is very conservative because only maximum variation is roughly considered as indicated in Figure 4.2b.

Thus, an effective robust design should be developed for these kind of systems.

In this chapter, a multi-domain modeling-based robust design approach is presented to design a nonlinear system to be robust under large parameter variations. Since this approach integrates the merits of both multi-domain modeling and robust design to handle the influence of system nonlinearity as well as large parameter variations, it can effectively ensure robustness of the nonlinear system even if large parameter variation exists. In this method, the large parameter region is first divided into many small subdomains. At each small subdomain, the complexity of the nonlinear system will be reduced so that the system can be approximated by a linear model. Thus, a robust design method is developed to minimize variation influence at all subdomains in relation to design performance. Finally, several examples are

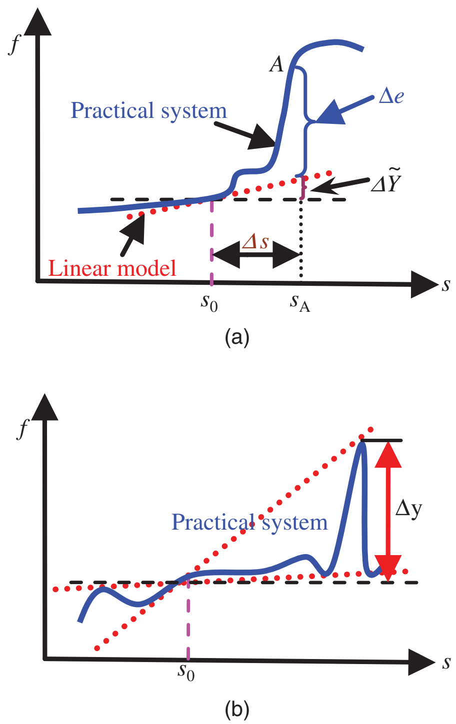


FIGURE 4.2 Robust design for the nonlinear system: (a) linear model approximation; (b) variable sensitivity model approximation

conducted to demonstrate and confirm the effectiveness of the presented method in comparison with two common design methods.

4.2 MULTI-DOMAIN MODELING-BASED ROBUST DESIGN METHODOLOGY

It is well known that the system is easy to handle if its model has a well linear structure. In general, a well-developed system model should have a satisfactory performance near its working condition. In order to have a robust performance over the larger working area, an effective design approach should consider the influence of both nonlinear term and large parameter variation simultaneously. Thus, for the nonlinear design problem (Equation 4.1), a multi-domain modeling-based robust design approach is developed, as indicated in Figure 4.3. Since this approach integrates the merits of both multi-domain modeling and robust design, it can effectively ensure robustness of the nonlinear system in a large parameter variation domain.

In this method, the parameter region is first divided into many small subdomains, so that the nonlinear system at each subdomain can be approximated by

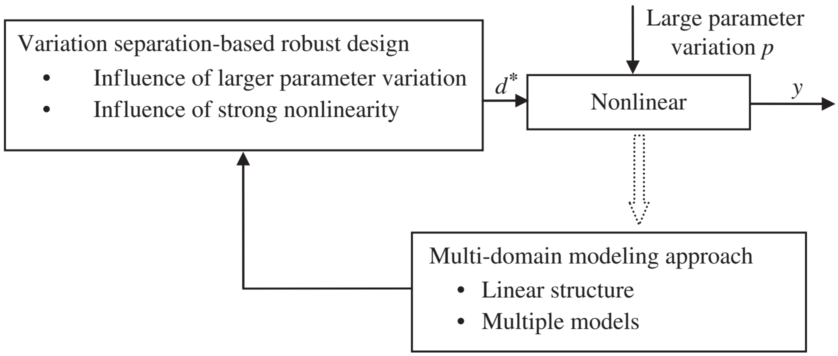


FIGURE 4.3 A multi-domain modeling-based robust design method

a linear model. Then, a robust design is developed to minimize the influence of parameter variation at all these subdomains. The proposed method has the following advantages:

- It has a high modeling precision since the method considers the influence of the system nonlinearity in a large parameter region.
- It can be easily handled by well-developed robust design theories, since the model built owns a well linear structure.
- It can make a nonlinear system to be robust against large parameter variation.

4.2.1 Multi-Domain Modeling Approach

Most robust design approaches use the linearization model built at the nominal value p_0 to design the system robustness since the model obtained is simple and easily handled by the developed robust design method. However, when the system is strongly nonlinear and has a large parameter variation, it is well known that the linearization model built at the nominal value p_0 is difficult to describe this system well due to large approximation error. Alternatively, a multi-domain modeling approach is often used to model the nonlinear system in the control field. This method has a well linear structure and satisfactory modeling performance. However, it is never applied in robust design of nonlinear system under large parameter variation.

A simple example for single parameter and single performance is indicated in Figure 4.4 for the explanation of the multi-domain modeling method. The whole parameter region is divided into many small subdomains, and the system at each subdomain is approximated by a local linearization model around its center point. If the subdomain is small enough, its local linearization model can very well describe the behavior of the system at this subdomain. Thus, this modeling approach has the following advantages: (a) using a group of linear models, it can express the nonlinear system well within a large parameter domain; (a) the model built has a well linear structure.

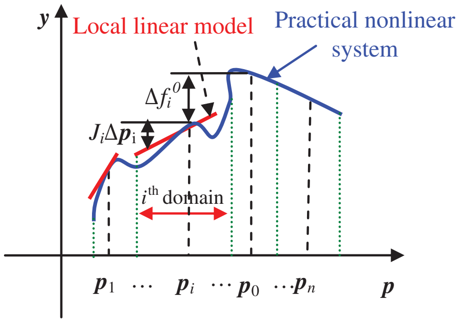


FIGURE 4.4 Multi-domain modeling approach

Since the whole domain is first divided into n small subdomains, for p at the ith subdomain, the corresponding performance of the system can be approximated by

$$y_i = f_i(d, p_i) + J_i \Delta p_i \tag{4.2}$$

with $J_i = \left.\frac{\partial f}{\partial p}\right|_{p=p_i}$ and $\Delta p_i = p - p_i$.
where p_i is the center of the ith domain and $f_i(d, p_i)$ is the performance value at its center.

Define

$$\Delta y_i = y_i - f(d, p_0) \text{ and } \Delta f_i^0 = f_i(d, p_i) - f(d, p_0) \tag{4.3}$$

From Equations 4.2 and 4.3, we have

$$\Delta y_i = \Delta f_i^0 + J_i \Delta p_i \tag{4.4}$$

From Equation 4.4, it is clear that the performance variation Δy_i at each subdomain is decided by two parts: central variation Δf_i^0 and domain variation $J_i \Delta p_i$, as indicated in Figure 4.2. The central variation represents the distance between the nominal value p_0 and the center of every domain, and the domain variation represents the performance variation within each domain around its center. Thus, minimization of these two variations Δf_i^0 and $J_i \Delta p_i$ are required to achieve the robustness in each subdomain.

4.2.2 Variation Separation-Based Robust Design Method

From Equation 4.4, the sum of all performance variations on all domains may be written as

$$\Delta Y = \sum_{i=1}^{n} \left\| \Delta f_i^0 + J_i \Delta p_i \right\| \tag{4.5}$$

with $\Delta Y = \sum_{i=1}^{n} \|\Delta y_i\|$

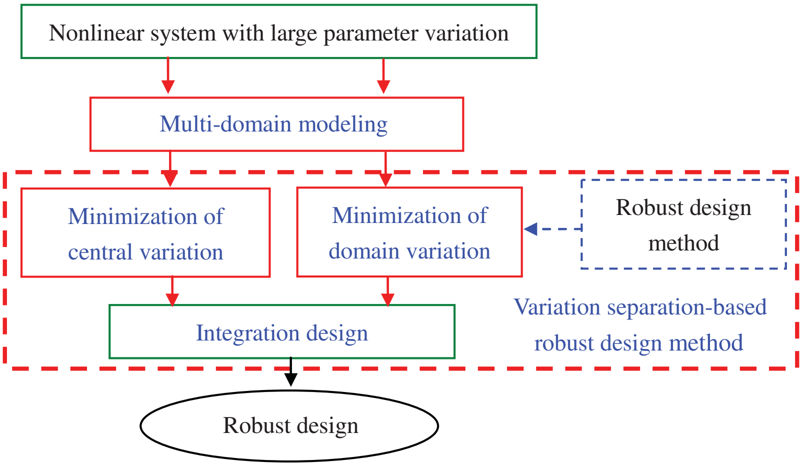


FIGURE 4.5 Variation separation-based robust design

When this variation sum (Equation 4.5) is minimized, then the nonlinear system is robust. However, it is difficult to directly minimize this variation sum since it is affected by two coupling parts: the central variation and the domain variation, and these two parts have the nonlinear relation with design variable d.

According to the inequality theory, the equality (Equation 4.5) may be expressed as

$$\Delta Y \leq \sum_{i=1}^{n} \left\| \Delta f_i^0 \right\| + \sum_{i=1}^{n} \left\| J_i \Delta p_i \right\| \tag{4.6}$$

From this design problem (Equation 4.6), if the upper bound of the performance variation is minimized, then the system will be robust even if the large parameter variation exists.

Thus, a variation separation-based robust design method is developed, as shown in Figure 4.5. First, the central variation and the domain variation are separated, upon which the complex design will be reduced into two simple subtasks. One is to minimize the sum of all central variations. The other is to minimize all domain variations using the robust design method. Then, to simultaneously consider their effects, these two subtasks are integrated into a unified optimization framework to ensure the robustness of the nonlinear system under circumstance of large parameter variations.

4.2.2.1 Minimization of the Central Variation First, the first term $\sum_{i=1}^{n} \left\| \Delta f_i^0 \right\|$ on the right side of the inequality (Equation 4.6) is only controlled by centers of all subdomains and has no relationship to parameter variation Δp. Since all domains' centers need to be close to the performance nominal value, the design variable d should be chosen to minimize all $\Delta f_i^0 (i = 1, \ldots, n)$ at all domains

$$C_1(\mathrm{d}): \quad \begin{array}{c} \min\limits_{d} \frac{1}{n} \sum\limits_{i=1}^{n} \left\| \Delta f_i^0 \right\|_2 \\ \text{s.t. } d \in S_d,\ p \in S_p \end{array} \tag{4.7}$$

The solution of C_1(d) can make all domains' centers close to the performance nominal value.

4.2.2.2 *Minimization of the Domain Variation*

The domain variation can be reduced by minimizing other term $\sum_{i=1}^{n} \|J_i \Delta p_i\|$ on the right side of the inequality (Equation 4.6). Define

$$\Delta r_i = J_i \Delta p_i \tag{4.8}$$

From Equation 4.8, we have

$$\left\| \Delta r_i \right\|_2^2 = \Delta p_i^T J_i^T J_i \Delta p_i \tag{4.9}$$

According to the singular value decomposition theory, we have

$$\left\| \Delta r_i \right\|_2^2 = \sum_{j=1}^{m} \sigma_{i,j} w_{i,j}^2 \tag{4.10}$$

with $[w_{i,1}, \ldots, w_{i,m}]^T = V_i^T \Delta p_i$.
where $\sigma_{i,j}$ is the jth singular value of $J_i^T J_i$, and its orthogonal eigenvector is denoted as $V_i = [V_{i,1} \ \cdots \ V_{i,m}]$.

According to the sensitivity-based robust design approach, only if the maximal singular value $\sigma_{i,\max}$ is minimized, the performance Δr_i will be less sensitive to the parameter variation Δp_i. For the robustness of the whole system, all maximal singular values at all subdomains should be minimized. This can make every subdomain to have small variation around its center. Thus, in order to minimize Δr in the whole parameter region, together with minimization of $\sum_{i=1}^{n} \|J_i \Delta p_i\|$, the design variable d should be chosen to minimize all maximal singular values $\sigma_{i,\max} (i = 1, \ldots, n)$

$$C_2(\mathrm{d}): s.t. \quad \begin{array}{c} \min\limits_{d} \frac{1}{n} \sum\limits_{i=1}^{n} \max\limits_{j=1,\ldots,m} (\sigma_{i,j}) \\ \left\| \Delta r_i \right\|_2^2 = \sum\limits_{j=1}^{m} \sigma_{i,j} w_{i,j}^2 (i = 1, \ldots, n) \\ d \in S_d, p \in S_p \end{array} \tag{4.11}$$

The solution of Equation 4.11 can minimize the influence of the domain variation, which means that performance at every subdomain is close to the performance at the center of the domain.

4.2.2.3 Integrated Design This above designs will be integrated to simultaneously consider the influence of the central variations and the domain variations. This will make the performance of the designed system close to its nominal performance value and, thus, it can ensure the robustness of the nonlinear system even if large parameter variation exists. In this sense, a multi-objective optimization is constructed for the integration design.

$$\begin{cases} \min\limits_{d} \ C_1(\mathrm{d}) \\ \min\limits_{d} \ C_2(\mathrm{d}) \\ \end{cases} \qquad (4.12)$$
$$s.t. \quad d \in S_d,\ p \in S_p$$

The most common method to solve the multi-objective optimization is the weighted-sum (WS) method, which optimizes the weighted sums of several objectives as below:

$$\min_{d} \beta \frac{C_1(\mathrm{d})}{C_1(\mathrm{d}^+)} + (1-\beta)\frac{C_2(\mathrm{d})}{C_2(\mathrm{d}^+)} \qquad (4.13)$$
$$s.t. \quad d \in S_d,\ p \in S_p$$

where the objectives $C_1(\mathrm{d})$ and $C_2(\mathrm{d})$ are normalized by their central values $C_1(\mathrm{d}^+)$ and $C_2(\mathrm{d}^+)$ with the central point d^+ of d, and β is a trade-off weight in the range $0 \prec \beta \prec 1$.

Obviously, the robust design problem can be solved from Equation 4.13 with a proper choice of the weight factor β. In order to obtain this proper weight factor β, an intelligent approach is presented for optimal solution, as shown in Figure 4.6. It mainly includes the following steps:

Step 1: Initialize the weight factor β.

Step 2: Solve the optimal problem (Equation 4.13).

Step 3: Check the solution to make sure whether it satisfies the robust performance. If not, this weight factor β needs to be updated. Here, the well-developed particle swarm optimization method is employed to update the weight factor β and then the program goes to Step 2.

Step 4: Repeat Steps 2 and 3 until the design satisfies the robustness criteria.

4.2.3 Design Procedure

In industrial application, many systems have strong nonlinearity as well as large parameter variation. The presented method is to ensure the robustness of this kind of systems and is summarized in Figure 4.7. Since it integrates the merits of both

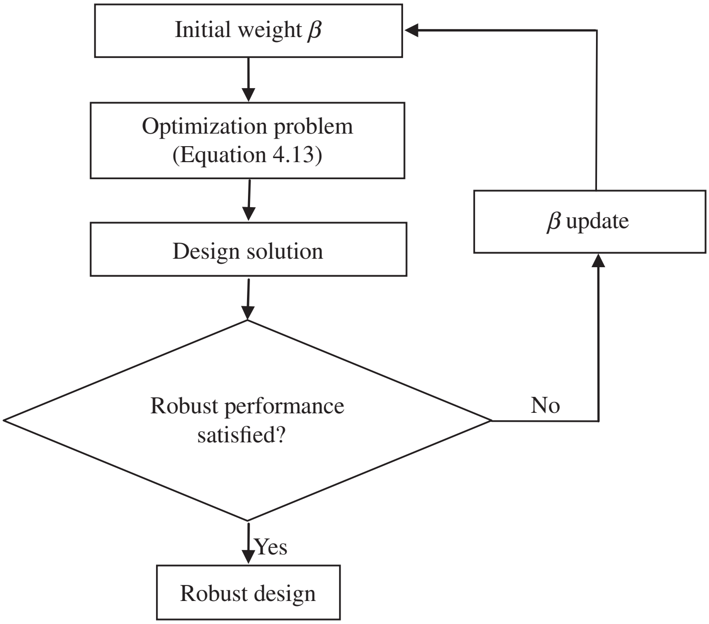


FIGURE 4.6 Configuration for intelligent optimization

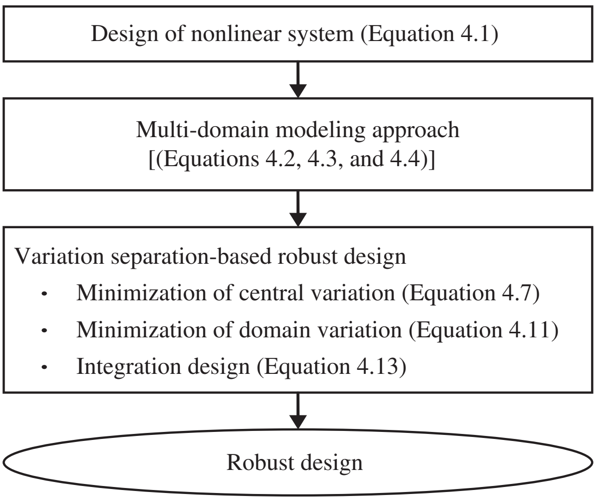


FIGURE 4.7 Design flowchart

multi-domain modeling and robust design to handle the influence of system nonlinearity as well as large parameter variation, it can effectively ensure robustness of the nonlinear system even if large parameter variation exists.

Obviously, when there is only one subdomain, the above presented approach will be the same with the traditional robust design method that is based on the linearization model at the nominal value, such as Euclidean norm method, conditional number method, and first- and second-order moment methods. These traditional robust design methods can be considered as a special case of the presented approach.

4.3 CASE STUDY

Two practical nonlinear cases are used to demonstrate the effectiveness of the proposed design method. Define the relative modeling error RE as

$$RE(i) = \frac{\left|y^i - \widehat{y}^i\right|}{|y^i|} \times 100\% \tag{4.14}$$

where y and $\widehat{y}$ represent the practical performance and the estimated performance respectively, and i refers to the ith sample.

4.3.1 Robust Design of a Belt

Belts are used in transmission of power between shafts with either parallel or skewed axes. The power transmitted by a belt is

$$W = f(s) \tag{4.15}$$

with $f(s) = (1 - e^{-\xi\theta})(T - MV^2)V$

where M and θ are the mass of the belt per unit length and the contact angle respectively, V and T are the belt speed and the tension in the belt, respectively, ξ and W are the coefficient of friction and the transmitted power, respectively. The coefficient of friction ξ, the mass M, and the contact angle θ are 0.5, 1 kg, and $\pi/4$, respectively. The design variable d, the design parameter p, and the performance function y are

$$d = T,\ p = V,\ y = W$$

This design task is to find the design variable T from the design space [10, 22] to have a robust performance against variation $\Delta p = \Delta V$.

4.3.1.1 Comparison of Modeling Performance

The modeling performance is first compared using 1000 samples. Since the design parameter V varies within [0.8, 2.4], it is divided into four subdomains, each with the interval equal to 0.4 at the proposed method. From Figure 4.8, it is clear that the proposed method has smaller

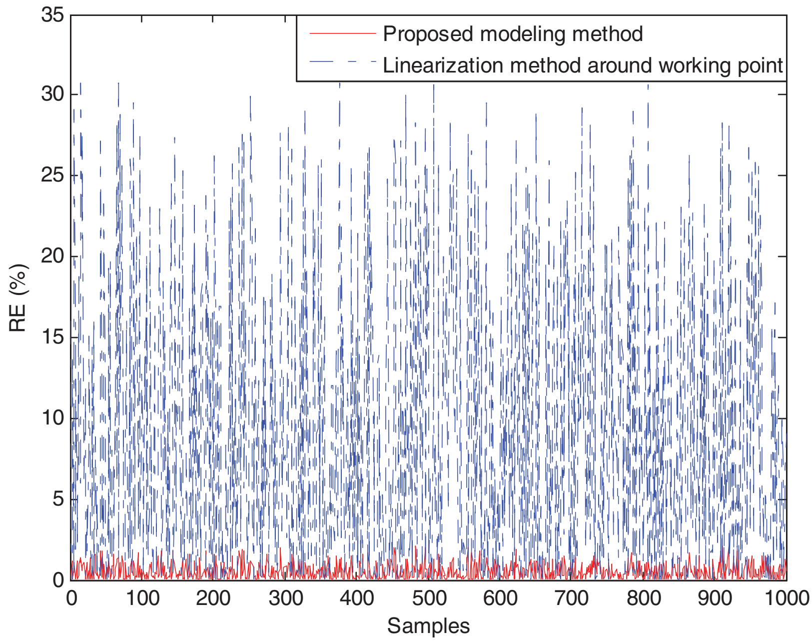


FIGURE 4.8 Comparison of relative errors: proposed method—(mean: 0.541, variance: 0.236); linearization method—(mean: 8.68, variance: 75.27)

relative error, because the largest *RE* obtained by the proposed method is 2% but the one obtained by linearization method around the nominal parameter is 30%, and its mean and variance are also smaller than the linearization method. Thus, the proposed method has a better modeling performance than the traditional method because it considers system nonlinearity.

4.3.1.2 Comparison of Design Performance After the above modeling, the design performance is compared in Table 4.1 under different design methods and large parameter variation. From Table 4.1, it is clear that the mean and the variance of the performance variation Δy gained by the proposed method are smaller than the other two common methods: Euclidean norm method (Ting and Long, 1996; Zhu and Ting, 2001) and sensitivity-region-based robust design (Li, Azarm, and Boyars, 2006).

Moreover, according to six sigma theory in quality engineering (Chen et al., 1996, 1999), 99.73% of variations will fall into the range of mean $\pm$ 3 standard deviations, under assumption that the variation is normally distributed. Thus, the smaller range of mean $\pm$ 3 standard deviations, the better robustness that the method has. Obviously, the proposed method has better robustness than the other two methods.

4.3.2 Robust Design of Hydraulic Press Machine

The forging workings of the hydraulic press machine are shown in Figure 4.9. Since its working plate has a great inertia and forged work piece has ultrahigh strength and

TABLE 4.1 Performance Comparison Under Different Design Methods

		Linearization-Based Robust Design Method	Sensitivity-Region-Based Robust Design Method	Proposed Robust Design Method
	Design d^*	15.4	16.6	16.48
ΔY	Mean	−0.6591	−0.6585	−0.6585
	Variance	0.3660	0.3565	0.3546
	Mean ± 3 standard deviation	[−1.7572, 0.4390]	[−1.7282, 0.4111]	[−1.7224, 0.4053]

is big in size, it needs a huge driving force to forge a metal work piece and to propel the working plate to move up and down. This calls for a complex hydraulic driven system, including a pump, valve, pipe, and cylinder. The objective of this design is to achieve robustness of the dynamic process under parameter variation, because it is easy to estimate and predict system performance when the dynamic process is robust.

According to Newton's second law, the force model of the working plate can be derived as

$$m\frac{d^2x}{dt^2} = AP - B_c\frac{dx}{dt} + mg - f \tag{4.16}$$

where m is mass of the working plate, x is position of the working plate, A is area sum of all hydraulic cylinders, P is pressure, f is load force, and B_c is viscous damping coefficient.

The flow model of the hydraulic cylinder can be represented by

$$q = A\frac{dx}{dt} + \frac{V_0}{\beta_e} \cdot \frac{dp}{dt} \tag{4.17}$$

where q is flow, V_0 is initial volume that is equal to the product of the cylinder area and the piston position h, and β_e is the spring moment of medium.

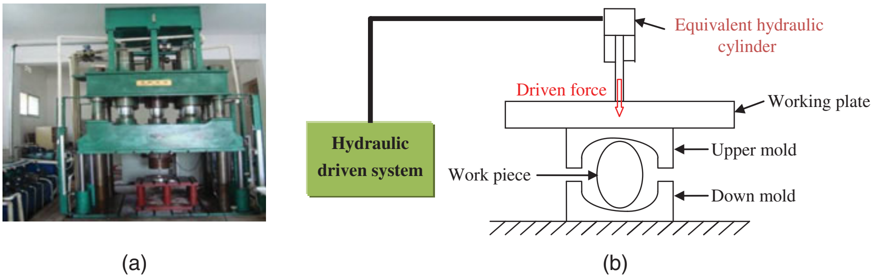


FIGURE 4.9 The hydraulic press machine system and its operation principle: (a) hydraulic press machine; (b) forging process

The pressure P can be divided into two parts:

$$P = P_0 + \Delta P \tag{4.18}$$

where P_0 and ΔP are the nominal pressure and the pressure variation, respectively, nominal pressure is used to balance both weight and load force, and pressure variation is used to offer the desired dynamic trajectory. Since displacement during the forging process is small and its speed is also low, the nominal pressure P_0 that is usually known can be estimated as follows

$$AP_0 + mg - f = 0 \tag{4.19}$$

Inserting Equations 4.18 and 4.19 into Equations 4.16 and 4.17, we have

$$m\frac{d^2x}{dt^2} = A\Delta P - B_c\frac{dx}{dt} \tag{4.20}$$

$$q = A\frac{dx}{dt} + \frac{V_0}{\beta_e} \cdot \frac{d\Delta P}{dt} \tag{4.21}$$

Operating Equations 4.20 and 4.21 using Laplace transform and inserting Equation 4.21 into Equation 4.20, the system model can be obtained as follows:

$$v = \frac{dx}{dt} = \frac{1}{\dfrac{s^2}{\omega^2} + \dfrac{2\xi}{\omega}s + 1}u \tag{4.22}$$

where v is speed, $u = \frac{q}{A}$ and

the frequency:

$$\omega = \sqrt{\frac{\beta_c A}{hm}} \tag{4.23a}$$

the damping ratio:

$$\xi = \frac{B_e}{2}\sqrt{\frac{h}{m\beta_e A}} \tag{4.23b}$$

From Equation 4.22, it is clear that the dynamic performance of this system is the function of its frequency ω and damping ratio ζ. Thus, the dynamic robust performance may be transformed equivalently into robustness of these two characteristic variables. Since the parameter $p = \begin{bmatrix} A & \beta_e \end{bmatrix}$ has variation Δp around its nominal value p_0, the design variable $d = h$ is required to choose from the design space $h \in [0.5, 5]$ to ensure the robustness of these two characteristic variables, in order to guarantee the dynamic robustness of the system under parameter variation.

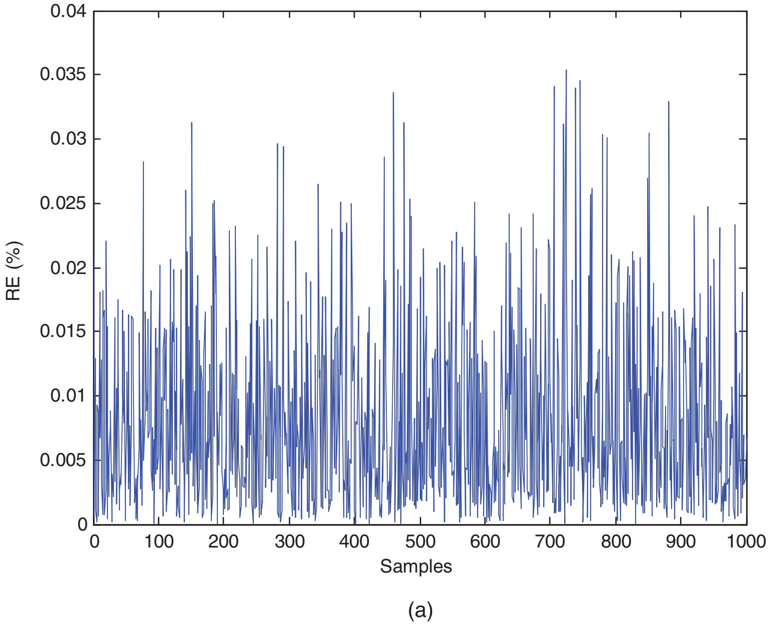


FIGURE 4.10 Comparison of relative error: (a) relative error of the presented method (mean: 0.0077, variance: 4.53×10^{-5}); (b) relative error of the common linearization method (mean: 0.0165, variance: 1.22×10^{-4})

TABLE 4.2 Mean and Variance of Modeling Error

	Modeling Error	
	Mean	Variance
Presented method	5.53×10^{-6}	2.34×10^{-11}
Linearization method	3.45×10^{-5}	5.17×10^{-10}

4.3.2.1 Comparison of Modeling Performance This design parameter p randomly varies in $[-5\%p_0, 5\%p_0]$ and is divided into nine subdomains. The modeling performance of this presented approach is compared to the linearization method around the nominal point under 1000 samples.

From Figure 4.10, it is clear that the presented method has smaller relative error, because the largest RE obtained by the presented method is 0.035%, and the one obtained by the linearization method is 0.05%. Furthermore, the mean and variance obtained in the proposed method are also smaller than the linearization method. From Table 4.2, the modeling error, including its mean and variance, is also smaller than the one obtained by the linearization method around the nominal parameter. Thus, the presented method has better modeling performance than the linearization method, because it has considered the system nonlinearity.

4.3.2.2 Comparison of Design Performance The comparison of the design performance under different design methods is shown in Table 4.3 when large parameter variation exists. From Table 4.3, it is clear that the mean and variance of the performance variation Δy gained by the presented method are smaller than the performance variation Δy gained by the other two common methods: Euclidean norm method (Ting and Long, 1996; Zhu and Ting, 2001) and sensitivity-region-based robust design (Li, Azarm, and Boyars, 2006).

Moreover, according to six sigma theory in quality engineering, 99.73% of variations will fall into the range of mean ± 3 standard deviations, under assumption that the variation is normally distributed. It is obvious that the range of mean ± 3 standard deviation obtained by the presented method is smaller than the ones obtained by the

TABLE 4.3 Performance Comparison Under Different Design Methods

	$\Delta y = \lVert y(d, p_0)\rVert_2^2 - \lVert y(d, p_0 + \Delta p)\rVert_2^2$		
	Mean	Variance	Mean ± 3 Standard Deviation
Euclidean norm method	-5.31×10^{-6}	1.820×10^{-8}	$[-0.529, -0.532] \times 10^{-5}$
Sensitivity region method	4.178×10^{-6}	1.404×10^{-8}	$[0.419, 0.416] \times 10^{-5}$
Presented method	-7.77×10^{-7}	5.193×10^{-10}	$[-0.776, -0.778] \times 10^{-6}$

other two methods. Thus, the presented method has better robustness than the two common design methods.

4.4 SUMMARY

A design method is presented for robustness of a nonlinear system against large parameter variations. Since the parameter space is divided into many small subdomains, a local linearization model can be well developed at each subdomain. The nonlinear system can then be described well by the multi-domain modeling approach in a large parameter space. The variation separation-based robust design method developed can effectively minimize the influence of parameter variation on design performance. Since this presented method integrates the merits of both multi-domain modeling and robust design, it can ensure the robustness of the nonlinear system under large parameter variations. Effectiveness of the method is illustrated in comparison with the two common design approaches for designing a practical nonlinear system. The results show that it can obtain the desired robust performance and demonstrates its superiority over the traditional design methods when the system is strongly nonlinear and has large parameter variations.

CHAPTER 5

HYBRID MODEL/DATA-BASED ROBUST DESIGN UNDER MODEL UNCERTAINTY

The previous two chapters mainly discussed robust design under parameter variations. In this chapter, two hybrid model/data-based, probabilistic and deterministic, robust design approaches are proposed for a partially unknown system under parameter variations as well as model uncertainty. The system is first formulated into a linear structure that will be easy to handle by the well-developed robust design theories. Its sensitivity matrix incorporates all model uncertainties and nonlinearities. Then, the bound of the sensitivity matrix can be estimated from data. Since modeling of the bound is easier than direct modeling of the matrix perturbation, the two model-based robust design methods, one in deterministic nature and another in probabilistic nature, are developed to minimize the influence of parameter variation to the performance.

5.1 INTRODUCTION

In manufacturing process, many systems used are partially unknown due to various reasons. Model uncertainty could be caused by incomplete system information, simplification, and idealization at the design stage. If this model uncertainty is not properly considered at the design stage, it may cause failure in future operations. Thus, in order to achieve desirable robust performance, the influence of model uncertainty on performance should be minimized.

System Design and Control Integration for Advanced Manufacturing, First Edition. Han-Xiong Li and XinJiang Lu.

However, as presented in Chapter 2, since data-based robust designs do not rely on the process knowledge, they may lead to a relatively coarse solution and need large amounts of experimental data at high cost. On the other hand, model-based robust design approaches are not applicable to unknown systems because of the unavailability of accurate relationship between performance and design variables. This model uncertainty will be a major challenge to robust design. So far, there is still no effective work reported in the area of robust design to handle the partially unknown nonlinear system.

In this chapter, two hybrid model/data-based robust design methods, which integrate the advantages of the model-based robust design and the data-based modeling, are presented to achieve the robustness of the partially unknown nonlinear system. First, the system is formulated into a linear structure. Its sensitivity matrix, incorporating influences of model uncertainty and nonlinearity, is different from the common constant sensitivity matrix. With the help of this estimated sensitivity bound and the nominal model of the system, two model-based robust design methods, one in deterministic nature and another in probabilistic nature, are then developed to minimize the influence of parameter variation to performance. Finally, the presented methods are compared with several common robust design methods on the selected examples.

5.2 DESIGN PROBLEM FOR PARTIALLY UNKNOWN SYSTEMS

Robust design problem for the partially unknown nonlinear system is considered as:

$$Y = f(d, p) \tag{5.1}$$

where $Y = [y_1 \quad \cdots \quad y_m]^T$ represents performance vector; $d = [d_1 \quad \cdots \quad d_l]^T$ is adjustable design variable vector; $p = [p_1 \quad \cdots \quad p_n]^T$ denotes uncontrollable parameter vector; and $f(d,p) = [f_1(d,p) \quad \cdots \quad f_m(d,p)]^T$ includes two parts: known model information $f_0(d,p)$ and model uncertainty $\Delta f(d,p) = f(d,p) - f_0(d,p)$. This known model information, also called as the nominal model here, can be obtained from the process knowledge and the expert's experience, and model uncertainty is often caused by assumption and simplification at the design stage. For convenience, $f(d,p)$, $f_0(d,p)$, and $\Delta f(d,p)$ are simply denoted as f, f_0, and Δf.

5.2.1 Probabilistic Robust Design Problem

When the probabilistic information of Δp is available, the probabilistic robust designs can be derived to design system robustness. Generally, the parameter p follows Gaussian distribution with nonzero mean and nonidentical standard deviation. The expected value μ_p and the covariance matrix σ_p of the parameter variation Δp can be expressed as

$$\mu_p = E[\Delta p] \tag{5.2a}$$

$$\sigma_p = E[(\Delta p - \mu_p)(\Delta p - \mu_p)^T] = E[\Delta p \Delta p^T] - \mu_p \mu_p^T \tag{5.2b}$$

where the parameter variation Δp is defined as $\Delta p = p - p_0$, p_0 is the nominal value, and $E[\bullet]$ is the expected-value operator.

There are some difficulties in robustness design of the system (Equation 5.1) due to the following reasons.

1. unknown influence of the model uncertainty Δf to performance; and
2. simultaneous influence of both the uncontrollable variation Δp and the model uncertainty Δf.

Traditional probabilistic robust design (TPRD) method (Al-Widyan and Angeles, 2005) is to design system robustness based on its nominal model f_0 without consideration of model uncertainty. Taylor series expansion of the performance Y is usually taken at the nominal values p_0 as follows:

$$\Delta\tilde{Y} = J_0\Delta p \tag{5.3}$$

with the nominal sensitivity matrix $J_0 = \frac{\partial f_0(d,p)}{\partial p}\Big|_{p_0}$.

Obviously, from Equations 5.2a and 5.3, the expected value $\tilde{\mu}_{\tilde{Y}}$ of the performance variation $\Delta\tilde{Y}$ can be expressed as

$$\tilde{\mu}_{\tilde{Y}} = E[\Delta\tilde{Y}] = J_0\mu_p \tag{5.4}$$

and the corresponding covariance matrix $\tilde{\sigma}_{\tilde{Y}}$ of $\Delta\tilde{Y}$ is derived as

$$\tilde{\sigma}_{\tilde{Y}} = E[(\Delta\tilde{Y} - \mu_Y)(\Delta\tilde{Y} - \mu_Y)^T] \tag{5.5}$$

Inserting Equations 5.3 and 5.4 into Equation 5.5, the covariance matrix $\tilde{\sigma}_{\tilde{Y}}$ may be rewritten as

$$\begin{aligned}\tilde{\sigma}_{\tilde{Y}} &= E\left[J_0(\Delta p - \mu_p)(\Delta p - \mu_p)^T J_0^T\right] \\ &= J_0 \cdot E\left[(\Delta p - \mu_p)(\Delta p - \mu_p)^T\right] \cdot J_0^T\end{aligned} \tag{5.6}$$

Inserting Equation 5.2b into Equation 5.6, the covariance matrix $\tilde{\sigma}_{\tilde{Y}}$ is rewritten as

$$\tilde{\sigma}_{\tilde{Y}} = J_0\sigma_p J_0^T \tag{5.7}$$

Then, the TPRD method is to minimize the estimated covariance matrix $\tilde{\sigma}_{\tilde{Y}}$ through optimizing the design variable d.

Since the model uncertainty can lead to a larger error between nominal model (Equation 5.3) and practical system (Equation 5.1), this error may make the TPRD method less effective. As an example, at point A of Figure 5.1, the TPRD method uses the performance covariance $\tilde{\sigma}$ in Equation 5.7 to measure the system robustness. However, the practical covariance σ is very different with the estimated performance covariance $\tilde{\sigma}$ due to model uncertainty. The solid curve is denoted as the nominal

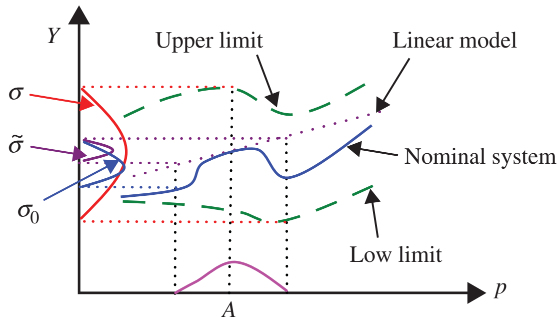


FIGURE 5.1 Disadvantage of the TPRD method

system and the dotted curve is represented as the upper and lower limits of the practical system in Figure 5.1.

Moreover, even if there is no model uncertainty, the linear approximation will produce large approximate error, which can also deteriorate system performance. For example, at design point A of Figure 5.1, the performance covariance $\tilde{\sigma}$ is different with the nominal performance covariance σ_0 of the nominal model f_0 due to the effect of the nonlinearity.

Thus, an effective probabilistic robust design approach should be developed to design robustness of the partially unknown nonlinear system.

5.2.2 Deterministic Robust Design Problem

When the probabilistic information of Δp is not available, the deterministic robust designs will be employed to design system robustness. Taking Taylor series expansion of Y at the nominal value p_0, the performance variation ΔY can be approximated by linear series expansion

$$\Delta Y = J \cdot \Delta d \tag{5.8}$$

where the nominal sensitivity matrix J_0, the perturbation sensitivity matrix ΔJ, and the sensitivity matrix J are defined as

$$J_0 = \left.\frac{\partial f}{\partial p}\right|_{p=p_0} \tag{5.9a}$$

$$\Delta J = \left.\frac{\partial \Delta f}{\partial p}\right|_{p=p_0} \tag{5.9b}$$

$$J = J_0 + \Delta J \tag{5.9c}$$

From Equation 5.8, the performance variation ΔY can be easily expressed as

$$\|\Delta Y\|_2^2 = \Delta p^T B \Delta p \tag{5.10}$$

with $\|\Delta Y\|_2^2 = \sum_{i=1}^{m} (\Delta y_i)^2$ and $B = J^T J$

Define

$$B_0 = J_0^T J_0 \tag{5.11}$$

According to the singular value decomposition (SVD) theory, the real symmetric matrices B and B_0 may be decomposed as

$$B = \zeta diag\left(\delta_1, \dots, \delta_n\right) \zeta^T \tag{5.12a}$$

$$B_0 = \zeta_0 diag\left(\delta_1^0, \dots, \delta_n^0\right) \zeta_0^T \tag{5.12b}$$

where δ_i and δ_i^0 are the singular values of J and J_0, respectively, and the corresponding orthogonal eigenvectors are denoted as ζ_i and ζ_i^0, which are one element of $\zeta = [\zeta_1 \quad \cdots \quad \zeta_n]$ and $\zeta_0 = [\zeta_1^0 \quad \cdots \quad \zeta_n^0]$.

The traditional deterministic robust design is to reduce the influence of the variations Δp based on the nominal model, and condition of $J = J_0$ and $B = B_0$. Thus, by inserting Equation 5.12b into Equation 5.10, the performance variations ΔY in the traditional robust method may be expressed as follows

$$\|\Delta Y\|_2^2 = \sum_{i=1}^{n} \delta_i^0 \left(x_i^0\right)^2 \tag{5.13}$$

with $[x_1^0, \dots, x_n^0]^T = \zeta_0^T \Delta p$.

Furthermore, the robust design variables d can be figured out by minimizing the largest singular value $\delta_{\max}^0$ of J_0 as well as the Euclidean norm method or minimizing the condition number $\frac{\delta_{\max}^0}{\delta_{\min}^0}$ of J_0 as well as the condition number method

$$C_1(d): \quad \begin{array}{c} \min\limits_d \max\left(\delta_i^0\right) \quad \text{or} \quad \min\limits_d \left(\frac{\delta_{\max}^0}{\delta_{\min}^0}\right) \\ st. \quad h(d,p) = 0 \\ \quad\quad l(d,p) \le 0 \end{array} \tag{5.14}$$

where h and l are constraints from other design aspects.

There exist two cases

- Case 1: There is no model uncertainty in the system, then $J = J_0$.
- Case 2: There is model uncertainty in the system, then $J \neq J_0$.

Obviously, the traditional deterministic robust design methods, including Euclidean norm method and condition number method, can work well in Case 1. However, in Case 2, since J is not equal to J_0 and B is not equal to B_0, there exists the difference $\Delta\delta_i$ between the singular values δ_i^0 and δ_i, as shown in Figure 5.2. This difference will result in the largest singular value $\delta_{\max}$ or the condition number $\frac{\delta_{\max}}{\delta_{\min}}$

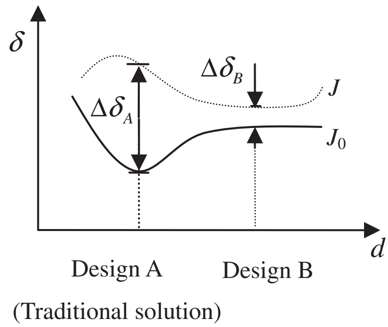


FIGURE 5.2 Influence of model uncertainty to singular value δ

of J not being minimal under the traditional robust design. Thus, the traditional robust design methods are less effective in this case. For example, design A in Figure 5.2 is a robust solution obtained by Euclidean norm method. However, its singular value will change significantly ($\Delta\delta_A$) due to the effect of ΔJ. Thus, it is not robust under model uncertainty. But the singular value δ_B in design B changes little under perturbation between J and J_0. Therefore, design B is less sensitive to the model uncertainty than design A.

If the singular value variation $\Delta\delta$ is very small, only the nominal variation δ^0 should be minimized so that the traditional deterministic robust design methods are still effective. Thus, the robust design problem under the model uncertainty can be decomposed into two subproblems. One is to reduce the influence of the variations Δp to the performance variations based on the nominal model. The other is to reduce influence of the model uncertainty to the variations of the singular values, which represents $\Delta\delta$ in Figure 5.2, so that the nominal singular value δ^0 is close to the singular value δ.

5.3 HYBRID MODEL/DATA-BASED ROBUST DESIGN METHODOLOGY

It is well known that the more the designer makes use of the system knowledge, the better the design can be achieved with fewer data samples required. Thus, to make better use of the system knowledge and reduce the uncertainty effect, the model-based robust design should be integrated with the data-based modeling approach for the partially unknown nonlinear system. This is because the model-based robust design can make full use of the known system information and the data-based modeling approach can effectively compensate uncertainty effect.

Based on this idea, a hybrid model/data-based robust design methodology is presented in Figure 5.3 to design robustness of the partially unknown nonlinear system. On one hand, the data-based uncertainty compensation method can extract useful information hidden in the data to compensate the effect of model uncertainty. On the other hand, with help of the data-based uncertainty compensation and the known

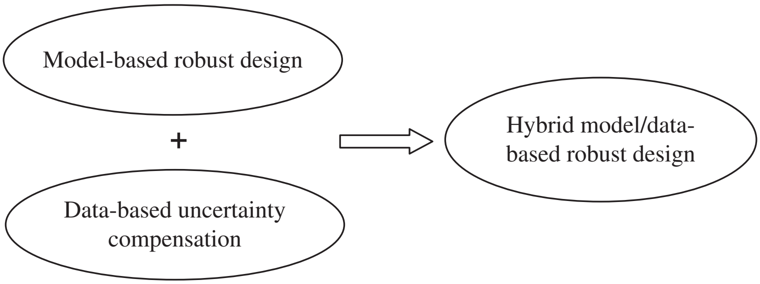


FIGURE 5.3 Hybrid model/data-based robust design methodology

nominal model, the model-based robust design approach can effectively minimize the influence of the parameter variation on the performance. Thus, this method integrates the advantages of both model-based robust design and data-based uncertainty compensation, and can be effectively applied to the partially unknown nonlinear system.

In the following two sections, two design methods, probabilistic robust design and deterministic robust design, will be discussed in detail.

5.3.1 Probabilistic Robust Design

5.3.1.1 Model Transformation for Linear Structure The most popular and mature sensitivity-based robust design methods, such as the condition number approach, the TPRD approach, and Euclidean norm approach, are based on a linear model. In order to make use of their advantages, the system is formulated into a linear structure. Taking Taylor series expansion of the nominal model f_0 at the nominal value p_0, the performance variation ΔY can be expressed as

$$\Delta Y = J_0 \cdot \Delta p + g(\Delta p) + \Delta f \tag{5.15}$$

with $J_0 = \left.\frac{\partial f_0}{\partial p}\right|_{p=p_0}$, and $g(\Delta p)$ includes all high order nonlinear terms of the nominal model f_0

Let

$$\Delta J \cdot \Delta p = g(\Delta p) + \Delta f \tag{5.16}$$

At the given design variable d, the meaning of Equation 5.16 can be illustrated approximately in Figure 5.4, where the perturbation sensitivity matrix ΔJ is expressed from the geometric relation. For example, ΔJ in point B is equal to the slope of the linear BO. Thus, this ΔJ incorporates the effect of all nonlinear terms as well as model uncertainty.

Since this ΔJ is a function of Δp, it varies under the given d, as illustrated in Figure 5.5, where solid curve denotes its mean and dotted curve represents the upper and lower limits.

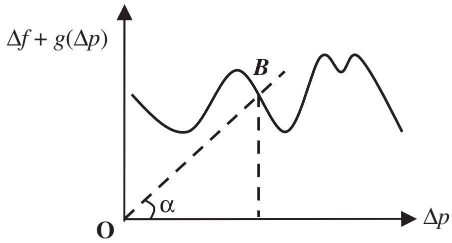


FIGURE 5.4 Meaning of perturbation matrix

From Equations 5.15 and 5.16, the performance variation ΔY can be rewritten as

$$\Delta Y = J\Delta p \tag{5.17}$$

with the sensitivity matrix $J = J_0 + \Delta J$.

Since the effect of the nonlinear terms and the model uncertainty are fully considered into ΔJ, the model (Equation 5.17) can well express the system (Equation 5.1). Moreover, the model (Equation 5.17) has a linear structure that would be easily designed by the model-based robust design approaches under condition that the sensitivity matrix J is known.

However, in Equation 5.17, only the nominal sensitivity matrix J_0 is known, while ΔJ is unknown due to unknown Δf. Thus, ΔJ needs to be identified from experimental data via Equation 5.17. Direct modeling of the perturbation sensitivity matrix ΔJ from data will be extremely difficult because: (a) its model structure is difficult to know, especially for strongly nonlinear systems; (b) sufficient sample data are difficult to obtain due to economic constraints; and (c) parameter variations are usually random. To reduce this difficulty, the bound model of the system could be used to provide some useful information of the system.

5.3.1.2 Data-Based Uncertainty Estimation In the bound model, the upper and lower bound of every element in the perturbation sensitivity matrix will be

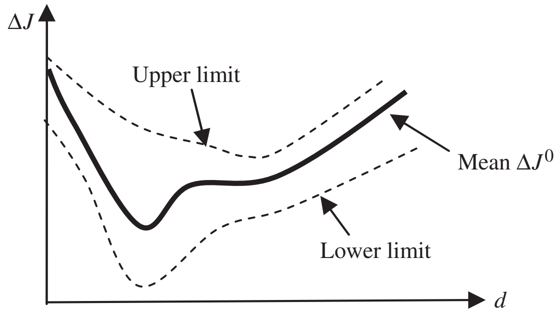


FIGURE 5.5 Perturbation sensitivity

estimated. The robust model of ΔJ at the given design variable d is mathematically represented as follows:

$$\Delta J = \begin{bmatrix} \Delta J_{1,1}^0 \pm \theta_{1,1} & \cdots & \Delta J_{1,n}^0 \pm \theta_{1,n} \\ \vdots & \ddots & \vdots \\ \Delta J_{m,1}^0 \pm \theta_{m,1} & \cdots & \Delta J_{m,n}^0 \pm \theta_{m,n} \end{bmatrix} \tag{5.18}$$

Given a design variable d, the element $\Delta J_{i,j}$ of ΔJ is limited between two extreme values, specified by $[\Delta J_{i,j}^0 - \theta_{i,j}, \Delta J_{i,j}^0 + \theta_{i,j}]$ with the mean value $\Delta J_{i,j}^0$ and bound value $\theta_{i,j}$.

For identification of Equation 5.18, let the parameter p randomly vary and then the sampling data of the performance variation ΔY in Equation 5.15 be collected under different design variable d. With the collected ΔY and the known nominal sensitivity matrix J_0, the sampling data of ΔJ can be obtained from Equation 5.17 at the given design variable d. Based on these sampling data of ΔJ, the mean value model of ΔJ can be identified from the least-square fitting, whereas the bound associated with each of the model elements can be calculated from the variance matrix (Choi and Allen, 2009).

Mean value model

Usually, a polynomial model is used to approximate the mean model

$$\Delta J^0 = g(x, \beta) \tag{5.19}$$

where g is a polynomial function, x is the polynomial vector about d, such as $x = [1\ d\ d^2]^T$, and β is the unknown coefficient vector of the polynomial. With the sampling data of the performance variation ΔY at hand, it is easy to identify the coefficient vector β using the common least-square method.

Bound estimation

This bound estimation can be found in Choi and Allen (2009) with some revisions. When ΔJ is not normal distribution, a suitable projection may transform it into the normal distribution. For simplification without loss of generalization, we assume that ΔJ is a normal distribution as

$$\Delta J_{i,j} \in N\left(\Delta J_{i,j}^0, \eta_{i,j}\right) \tag{5.20}$$

where η is the variance, which is usually modeled by an exponential function as

$$\eta_{i,j} = \exp(\gamma_{i,j} \cdot x) \tag{5.21}$$

Then, the unknown parameter $\gamma_{i,j}$ can be estimated by the following logarithm method as

$$\min_{\gamma_{i,j}} \sum_{i=1}^{N} \left[\log\left(\Delta J_{i,j} - \Delta J_{i,j}^{0}\right)^2 - \gamma_{i,j} \cdot x\right]^2 \tag{5.22}$$

Thus, from Equations 5.20 and 5.21, the bound for the mean model can be obtained using the interval estimation with a 100(1-a) confidence level:

$$\theta_{i,j} = t_{N-q,1-a/2} \exp(\gamma_{i,j} \cdot x) \tag{5.23}$$

where t refers to t distribution, and N and q are the number of sampling data and predictors, respectively.

5.3.1.3 Hybrid Model/Data-Based Robust Design After obtaining the bound model (Equation 5.18), the following task is to design the system robustness. Obviously, from Equations 5.2a and 5.17, the expected value μ_Y of the performance variation ΔY can be expressed as

$$\mu_Y = E[\Delta Y] = J\mu_p \tag{5.24}$$

and the corresponding covariance matrix σ_Y of ΔY is derived as

$$\sigma_Y = E\left[(\Delta Y - \mu_Y)(\Delta Y - \mu_Y)^T\right] \tag{5.25}$$

Inserting Equations 5.17 and 5.24 into Equation 5.25, the covariance matrix σ_Y may be rewritten as

$$\begin{aligned} \sigma_Y &= E[J(\Delta p - \mu_p)(\Delta p - \mu_p)^T J^T] \\ &= J \cdot E[(\Delta p - \mu_p)(\Delta p - \mu_p)^T] \cdot J^T \end{aligned} \tag{5.26}$$

Inserting Equation 5.2b into Equation 5.26, the covariance matrix σ_Y is rewritten as

$$\sigma_Y = J\sigma_p J^T \tag{5.27}$$

Then, the robust design is to minimize the covariance matrix σ_Y through optimization of design variable d. Since it is difficult to simultaneously minimize all elements of the covariance matrix σ_Y, a common method is to minimize its Frobenius norm.

According to the matrix theory (Al-Widyan and Angeles, 2005), the Frobenius norm of the covariance matrix σ_Y in Equation 5.27 may be expressed as

$$\|\sigma_Y\|_F \leq \sqrt{\frac{1}{m} tr(J^T J) tr(\sigma_p)} \tag{5.28}$$

where $||\cdot||_F$ is the Frobenius norm of $(\cdot)$. From Equation 5.28, it is clear that minimizing the performance covariance can be formulated as the minimization of its upper bound. Since the designer cannot control the term $tr(\sigma_p)\sqrt{\frac{1}{m}}$, this robust design problem reduces to the following minimization

$$\min_d tr(J^T J) \tag{5.29}$$

According to the matrix norm theory, $tr(J^T J)$ may be rewritten as

$$\frac{1}{\sqrt{m}} tr(J^T J) = \|J\|_F^2 = \sum_{i=1}^{n} \delta_i^2 \tag{5.30}$$

where δ_i is the singular value of J. The next task is to estimate these singular values. Define $B = J^T J$ and

$$B_0 = J_0^T J_0 \tag{5.31}$$

From Equations 5.17 and 5.31, the matrix B can be rewritten as

$$B = B_0 + J_0^T \Delta J + \Delta J^T J_0 + \Delta J^T \Delta J \tag{5.32}$$

The eigenvalues λ_i^0 $(i = 1, \ldots, n)$ of the matrix B_0 are related to the corresponding eigenvectors U_i^0 $(i = 1, \ldots, n)$ by the equation

$$B_0 U_0 = U_0 \Lambda_0 \tag{5.33}$$

where $U_0 = [U_1^0, \ldots, U_n^0]$ and $\Lambda_0 = diag(\lambda_1^0, \ldots, \lambda_n^0)$ are the right eigenvector set and the right eigenvalue set of B_0, respectively.

According to Bauer–Fike theorem (Stewart and Sun, 1990; Ralph and Stephen, 1989), if B_0 has an additive perturbation $\Delta B = J_0^T \Delta J + \Delta J^T J_0 + \Delta J^T \Delta J$, then a bound on the sensitivity of the eigenvalue is given by

$$|\lambda_i - \lambda_i^0| \leq K \cdot \|\Delta B\|_2 \tag{5.34}$$

where λ and λ^0 are the eigenvalue set of B and B_0, respectively, the condition number K is the ratio of the largest singular value $\sigma_{U\max}$ to the smallest singular value $\sigma_{U\min}$ of U_0, and $||\Delta B||_2$ is the Euclidean norm of ΔB.

According to the matrix norm theory, the norm matrix $||\Delta B||_2$ may be derived as

$$\|\Delta B\|_2 \leq (2\|J_0\|_2 + \|\Delta J\|_2) \cdot \|\Delta J\|_2 \tag{5.35}$$

Then, from the inequalities (Equations 5.34 and 5.35), we obtain

$$\left|\lambda_i - \lambda_i^0\right| \leq K \cdot \left(2\|J_0\|_2 + \|\Delta J\|_2\right) \cdot \|\Delta J\|_2 \tag{5.36}$$

According to the definition of the singular value, the following relation exists

$$\delta_i = \sqrt{\lambda_i}, \delta_i^0 = \sqrt{\lambda_i^0} \tag{5.37}$$

Thus, from Equations 5.36 and 5.37, the upper bound of δ_i can be expressed as

$$(\delta_i)^2 \leq \left(\delta_i^0\right)^2 + K \cdot \left(2\|J_0\|_2 + \|\Delta J\|_2\right) \cdot \|\Delta J\|_2 \tag{5.38}$$

In order to obtain the upper bound of δ_i given in Equation 5.38, the bound of $\|\Delta J\|_2$ is required to be estimated. Then, given the bound model Equation 5.18, the upper bound of $\|\Delta J\|_2$ can be obtained through the following optimization

$$\begin{aligned} \gamma(d) = & \max_{\Delta J_{i,j} \in \left[\Delta J_{i,j}^0 - \theta_{i,j}, \Delta J_{i,j}^0 + \theta_{i,j}\right]} \|\Delta J\|_2 \\ s.t. \quad & Eq.(5.18),\ Eq.(5.19) \text{ and } Eq.(5.23) \end{aligned} \tag{5.39}$$

where γ is the upper-bound value of $\|\Delta J\|_2$ under the given design variable d. The model $\gamma(d)$ can be obtained through the following three steps. First, identify the model (Equations 5.18, 5.19, and 5.23) using data; then obtain γ through the optimization (Equation 5.39) under a given design variable d using design of experiment (DoE), and finally build the model $\gamma(d)$ using RSM.

From Equations 5.38 and 5.39, the upper bound of δ_i can be rewritten as

$$(\delta_i)^2 \leq \left(\delta_i^0\right)^2 + K \cdot \left(2\|J_0\|_2 + \gamma(d)\right) \cdot \gamma(d) \tag{5.40}$$

Then, from Equations 5.29, 5.30, and 5.40, the robust design variable d can be figured out from the following optimization.

$$\begin{aligned} & \min_d \sum_{i=1}^{n} \left(\left(\delta_i^0\right)^2 + K \cdot \left(2\|J_0\|_2 + \gamma(d)\right) \cdot \gamma(d)\right) \\ & s.t. \quad B_0 U_0 = U_0 \Lambda_0 \end{aligned} \tag{5.41}$$

The optimization (Equation 5.41) can achieve the robustness of the nonlinear system under both model uncertainty and parameter variation.

Remark: Obviously, when the system is fully unknown, where the nominal singular value $(\delta_i^0)^2$ in Equation 5.40 is zero, the presented method becomes the experimental-data-based design method, such as meta-modeling method (Choi and Allen, 2009). When the model is fully known, where $K \cdot (2||J_0||_2 + \gamma(d)) \cdot \gamma(d)$ in Equation 5.40 is zero, the presented method becomes the model-based robust design method, such as

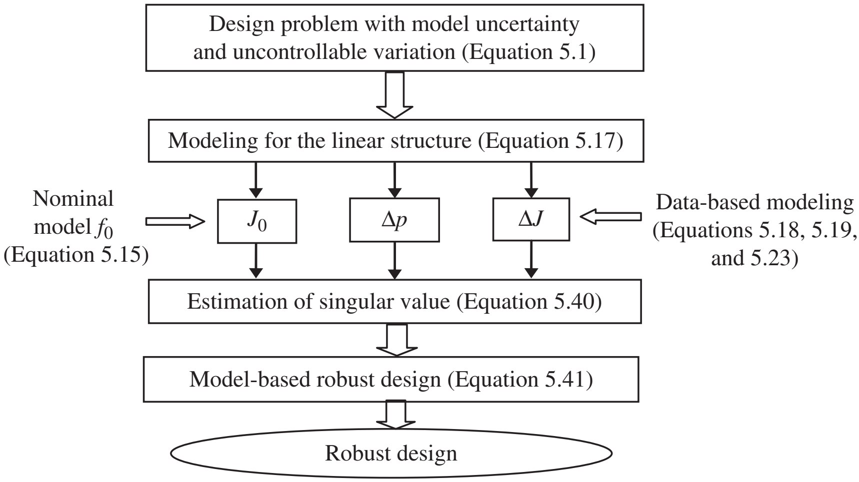


FIGURE 5.6 Design configuration

Euclidean norm method. When the model is partly known, this method is to integrate the advantages of these two methods to achieve the robustness.

5.3.1.4 Design Procedure In manufacturing applications, assumptions and idealization in a system often lead to model uncertainty. This model uncertainty is usually neglected by using the nominal model only for design and control. These designs derived from the nominal model only will be less robust because the model uncertainty neglected still affects the system performances. The proposed robust design method is to minimize the effect of the model uncertainty to the system performances.

The presented robust design methodology is summarized in Figure 5.6. The system is first modeled into a linear structure (Equation 5.17), where the perturbation sensitivity matrix ΔJ incorporates the effect of the nonlinear terms and model uncertainty (as Equation 5.16). Then, the data-based modeling approach is to estimate the model uncertainty from the process data (as Equations 5.18, 5.19, and 5.23). Finally, with the help of the estimated model and the known nominal model, the model-based method is developed to minimize the effect of the parameter variation to the performance (as Equation 5.41). Thus, the developed robust design approach can achieve the robustness of the nonlinear system under both the model uncertainty and the parameter variation.

5.3.2 Deterministic Robust Design

The perturbation bound S is used to estimate $\Delta\delta$ and defined as

$$\max\left(\left|\Delta\delta_i\right|\right) \leq S \tag{5.42}$$

with $\Delta\delta_i = \delta_i - \delta_i^0$

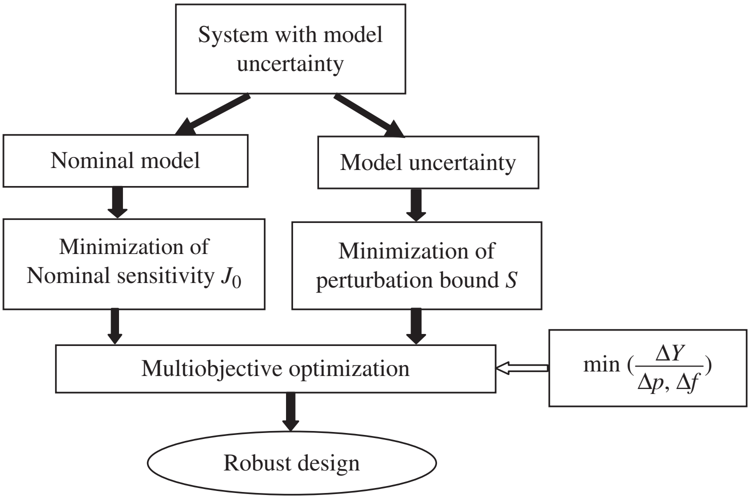


FIGURE 5.7 The new robust design methodology

If the perturbation bound S can be set very small by the selection of the suitable design variables d, then the singular value δ may be close to the nominal one δ^0. This means that the model uncertainty has small effect to the singular value δ. Under this condition, the traditional deterministic robust design methods are still effective.

Accordingly, these two subproblems can be transformed into two minimizations as shown in Figure 5.7. One is to minimize the nominal sensitivity matrix J_0 just as what the traditional methods do. This minimization can be achieved by solving the optimization problem C_1(d) in Equation 5.14. The other is to minimize the perturbation bound S. Then, based on these two minimizations, a multi-objective optimization problem is proposed to minimize the performance variations ΔY caused by both the model uncertainty Δf and the variations Δp, which is conceptually expressed as min $(\frac{\Delta Y}{\Delta p, \Delta f})$ in Figure 5.7.

5.3.2.1 Minimization of the Perturbation Bound S From the equality (Equation 5.10), matrix B can be rewritten as

$$B = B_0 + J_0^T \Delta J + \Delta J^T J_0 + \Delta J^T \Delta J \tag{5.43}$$

The eigenvalues $\lambda_i^0 (i = 1, \ldots, n)$ of the matrix B_0 are related to the corresponding eigenvectors $U_i^0 (i = 1, \ldots, n)$ by the equation

$$B_0 U_0 = U_0 \Lambda_0 \tag{5.44}$$

where $U_0 = [U_1^0, \ldots, U_n^0]$ and $\Lambda_0 = diag(\lambda_1^0, \ldots, \lambda_n^0)$ are the right eigenvector set and the right eigenvalue set of B_0, respectively.

According to Bauer–Fike theorem (Stewart and Sun, 1990; Ralph and Stephen, 1989), if B_0 has an additive perturbation $\Delta B = J_0^T \Delta J + \Delta J^T J_0 + \Delta J^T \Delta J$, then a bound on the sensitivity of the eigenvalue is given by

$$\left|\lambda_i - \lambda_i^0\right| \leq K \cdot \|\Delta B\|_2 \tag{5.45}$$

where λ and λ^0 are the eigenvalue sets of B and B_0, respectively; the condition number K is the ratio of the largest singular value $\sigma_{U\max}$ to the smallest singular value $\sigma_{U\min}$ of U_0.

According to the matrix norm theory, we have

$$\|\Delta B\|_2 \leq \left(2\|J_0\|_2 + \|\Delta J\|_2\right) \cdot \|\Delta J\|_2 \tag{5.46}$$

Then, from the inequalities (Equations 5.45 and 5.46), we obtain

$$\left|\lambda_i - \lambda_i^0\right| \leq K \cdot \left(2\|J_0\|_2 + \|\Delta J\|_2\right) \cdot \|\Delta J\|_2 \tag{5.47}$$

The inequality (Equation 5.47) may be rewritten as

$$\lambda_i^0 - K \cdot \left(2\|J_0\|_2 + \|\Delta J\|_2\right) \cdot \|\Delta J\|_2 \leq \lambda_i \leq \lambda_i^0 + K \cdot \left(2\|J_0\|_2 + \|\Delta J\|_2\right) \cdot \|\Delta J\|_2 \tag{5.48}$$

Since λ_i and λ_i^0 are the eigenvalues of $B = J^T J$ and $B_0 = J_0^T J_0$, respectively, λ_i and λ_i^0 are not smaller than zero. From the inequality Equation 5.48), if λ_i^0 is smaller than $K \cdot (2\|J_0\|_2 + \|\Delta J\|_2) \cdot \|\Delta J\|_2$, then the lower bound of λ_i can be negative, which is contradictory with $0 \leq \lambda_i$. In order to avoid such a case, the lower bound should take the maximal value between zero and $\lambda_i^0 - K \cdot (2\|J_0\|_2 + \|\Delta J\|_2) \cdot \|\Delta J\|_2$. Thus, the inequality Equation 5.48 becomes

$$max\left(\lambda_i^0 - K \cdot \left(2\,\|J_0\|_2 + \|\Delta J\|_2\,\right) \cdot \|\Delta J\|_2\,, 0\right) \leq \lambda_i \leq \lambda_i^0 + K \cdot (2\,\|J_0\|_2 + \|\Delta J\|_2) \cdot \|\Delta J\|_2 \tag{5.49}$$

where $max(\lambda_i^0 - K \cdot (2\|J_0\|_2 + \|\Delta J\|_2) \cdot \|\Delta J\|_2, 0)$ means that a maximal value between $\lambda_i^0 - K \cdot (2\|J_0\|_2 + \|\Delta J\|_2) \cdot \|\Delta J\|_2$ and zero is chosen.

According to the definition of the singular value, we have

$$\delta_i = \sqrt{\lambda_i}, \delta_i^0 = \sqrt{\lambda_i^0} \tag{5.50}$$

Thus, if the eigenvalues λ_i^0 and λ_i are very close, then their singular values δ_i^0 and δ_i are also very close. From the inequality (Equation 5.47) and the equality (Equation 5.50), if $K \cdot (2\|J_0\|_2 + \|\Delta J\|_2) \cdot \|\Delta J\|_2$ is very small, then

1. δ_i is close to δ_i^0.
2. $\Delta\delta_i$ is less sensitive to ΔJ.

Moreover, if $K \cdot (2\|J_0\|_2 + \|\Delta J\|_2) \cdot \|\Delta J\|_2 \approx 0$, then δ_i will be approximately equal to δ_i^0. Thus, minimizing the perturbation bound S can be transformed into the minimization of $K \cdot (2\|J_0\|_2 + \|\Delta J\|_2) \cdot \|\Delta J\|_2$ as

$$
\begin{aligned}
C_2(d)\colon & \min_d K \cdot (2\left\|J_0\right\|_2 + \|\Delta J\|_2) \cdot \|\Delta J\|_2 \\
& s.t.\ B_0 U_0 = U_0 \Lambda_0 \\
& h(d) = 0 \\
& l(d) \le 0
\end{aligned}
\tag{5.51}
$$

The solution of Equation 5.51 guarantees that $\Delta\delta_i$ is small and δ_i^0 is close to δ_i.

In the proposed robust design, only the bound of $\|\Delta J\|_2$ is required to be known. This bound can be easily estimated by experimental or simulation data, such as modeling method in Section 5.4.2. This proposed method can work well only if the variation of $\|\Delta J\|_2$ is limited within the estimated bound.

5.3.2.2 Multi-objective Optimization Multi-objective optimization is constructed to have the trade-off between two minimizations C_1 in Equation 5.14 and C_2 in Equation 5.51. The robust design variables d can be figured out from the following multi-objective optimization.

$$
\begin{aligned}
& \begin{cases} \min\limits_d C_1(d) \\ \min\limits_d C_2(d) \end{cases} \\
& S.t. \\
& \quad B_0 U_0 = U_0 \Lambda_0 \\
& \quad h(d,p) = 0 \\
& \quad l(d,p) \le 0
\end{aligned}
\tag{5.52}
$$

The most common method for the multi-objective optimization is the weighted-sum (WS) method, which optimizes the weighted sums of several objectives. The multi-objective optimization (Equation 5.52) can be easily derived by the WS methods as below:

$$
\begin{aligned}
& \min_d \beta \frac{C_1(d)}{C_1(d^+)} + (1-\beta) \frac{C_2(d)}{C_2(d^+)} \\
& S.t. \\
& B_0 U_0 = U_0 \Lambda_0 \\
& h(d,p) = 0 \\
& l(d,p) \le 0
\end{aligned}
\tag{5.53}
$$

where the objectives $C_1(d)$ and $C_2(d)$ are normalized by their central values $C_1(d^+)$ and $C_2(d^+)$ with the central point d^+ in the design variables space, and β is a trade-off weight in the range $0 \le \beta \le 1$.

The design variables d can be solved from Equation 5.53 with a proper choice of the weight factor β. In the proposed method, the weight β is selected according to the effect of the model uncertainty to performances. If the model uncertainty has smaller

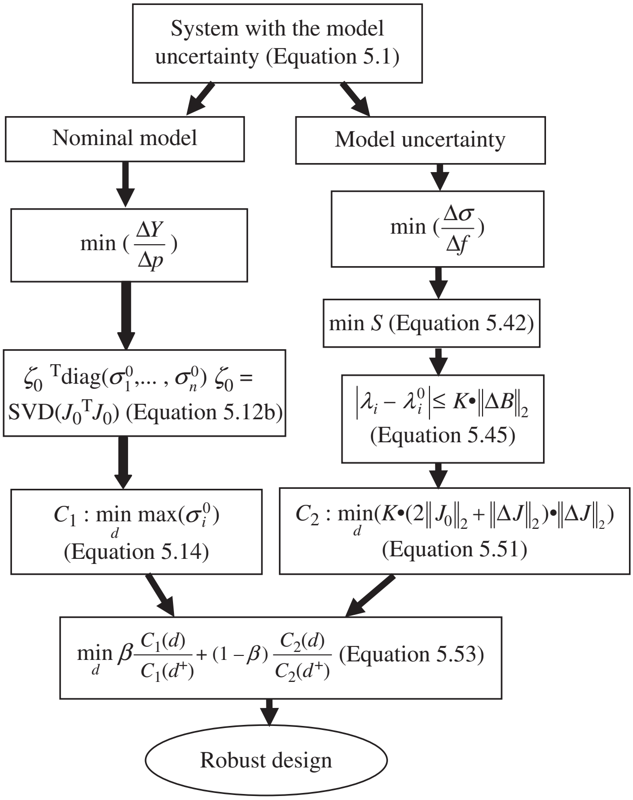


FIGURE 5.8 Design details of the proposed approach

influence to the performances, the objective function C_1 actually plays a bigger role. Then a big β should be chosen. When $\beta = 1$, it becomes the traditional deterministic robust design. On the other hand, if the model uncertainty has larger influence to the performances, the objective function C_2 would be more important and a smaller β should be chosen. When $\beta = 0$, it only minimizes the influence from the model uncertainty.

5.3.2.3 Design Summary The proposed robust design procedure is summarized in Figure 5.8, which is decomposed into two subproblems: one is to minimize the influence of the model parameter to the performance ($\Delta Y/\Delta p$) by using the nominal model only, and the other is to minimize influence of the model uncertainty to the performance ($\Delta\delta/\Delta f$) by making the nominal singular value δ_i^0 close to the singular value δ_i. Then, these two subproblems are solved respectively from the two following minimizations:

- The first optimization is to minimize the nominal sensitivity matrix J_0 just as what the traditional methods do. Using the singular value decomposition theory, minimizing the nominal sensitivity matrix J_0 can be transformed into minimizing the largest nominal singular value $\delta^0_{\max}$. Here, Euclidean norm is used as the robust index.

- The second optimization is to minimize the perturbation bound S. According to Bauer–Fike theorem, minimization of the perturbation bound S may be transformed into minimization of $K \cdot (2\|J_0\|_2 + \|\Delta J\|_2) \cdot \|\Delta J\|_2$.

A WS method can balance these two minimizations to make the performances less sensitive to both the model uncertainty and variations of the design variables. Since only the norm bound of the perturbation sensitivity matrix is needed, this proposed method can be easily realized.

5.4 CASE STUDY

5.4.1 Probabilistic Robust Design

Two cases are employed to compare the presented robust design method with the TPRD method. The performance index E_r is defined as

$$E_r = \|Y_p - \mu_p\|_2 - \|Y_T - \mu_T\|_2 \tag{5.54}$$

where Y_p and Y_T are performances gained by the presented method and the TPRD method, respectively, and μ_p and μ_T are their means. Since the difference $\|Y - \mu\|_2$ between the performance Y and its mean μ can estimate the degree of the deviation from its mean, the performance index E_r would be a better indication. Only if those with $E_r < 0$ is larger than 50%, then the robustness is improved with better performance than the TPRD method. Otherwise, the TPRD method performs better.

Example 5.1: Robust design for a pneumatic cylinder The design problem of a pneumatic cylinder is shown in the Example 1.2 in Chapter 1. The distance L, the pressure P, the nominal mass m_0, the nominal friction force F_0, and w are 1 m, 1.5 N/m^2, 1 kg, 0.01 N, and 0.001 N, respectively. The design variable d is chosen from [0.2 m, 0.3 m] to make the system robust to model uncertainty and parameter variations. ■

The nominal sensitivity matrix J_0 is

$$J_0 = \left[-\frac{1}{2m}\sqrt{\frac{gl(\pi D^2 P - 4F)}{2m}} \quad -\sqrt{\frac{2gl}{m(\pi D^2 P - 4F)}} \right] \tag{5.55}$$

Then, bound model (Equation 5.18) of the perturbation sensitivity matrix $\Delta J = [\Delta J_1, \Delta J_2]$ is estimated from data with the confidence level $a = 99.73\%$, where the perturbation sensitivity matrix ΔJ is limited in the bound, as shown in Figure 5.9. Given the nominal sensitivity matrix J_0 and the estimated model of the perturbation sensitivity matrix ΔJ, the approximate ability of the model (Equation 5.17) to the practical system (Equation 1.2) is shown in Figure 5.10, from which it is clear that

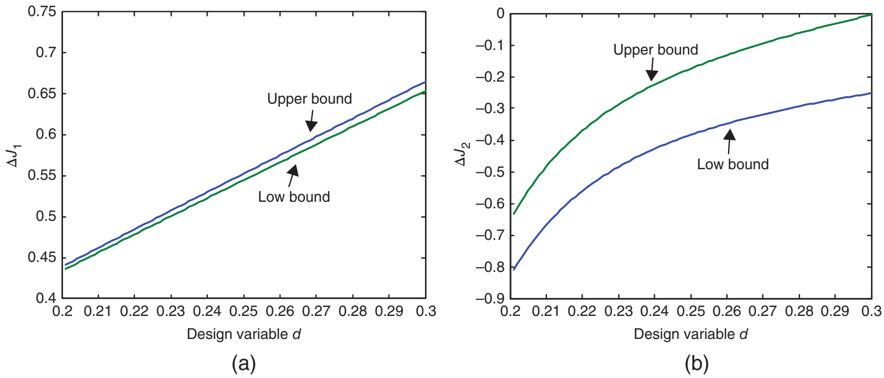


FIGURE 5.9 Data-based estimation for ΔJ: (a) estimation of ΔJ_1: (b) estimation of ΔJ_2

almost all performance variations are limited in the bound. Thus, it has a good capability to estimate the nonlinear system (Equation 1.2). It also shows that bound modeling (Equation 5.18) is much easier than direct modeling of all data points (circle points in Figure 5.10), because the sampling data have both strong nonlinearity and random nature.

Based on the obtained model (Equation 5.17), the presented robust design solution is $D_p = 0.3m$ through the optimization (Equation 5.41). The TPRD method is $D_T = D_{\min} = 0.2m$.

Verification is carried out for comparison. Assume that Δm and ΔF follow the normal distribution with mean $\mu_m = 0.05$, $\mu_F = 0.005$ and variance $\sigma_m = 0.01$, $\sigma_F =$

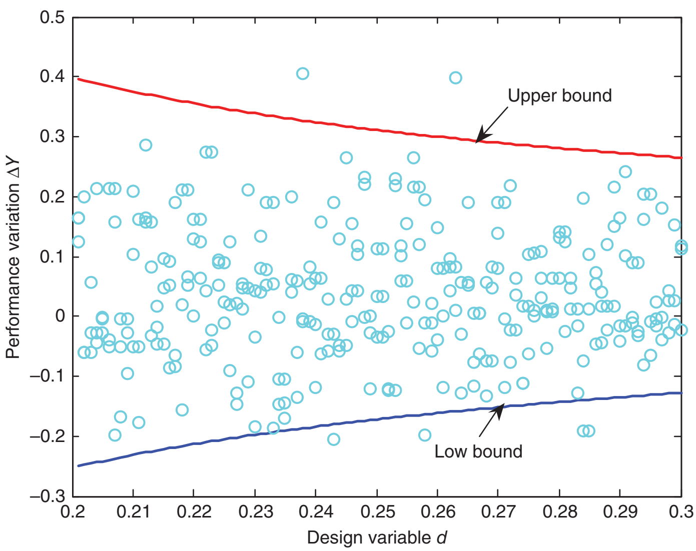


FIGURE 5.10 Bound modeling for the performance variation ΔY

TABLE 5.1 Performance Comparison Under Parameter Variations and Model Uncertainty

Performance V	TPRD Method	Presented Method
Variance σ_V	0.0153	0.0109

0.0001, respectively. A total of 1000 samples are taken for comparison of the variance σ_V with respect to Δm, ΔF, and the model uncertainty. From Table 5.1, we can see that the variance σ_V gained by the presented method is smaller than that obtained by the TPRD method. As shown in Figure 5.11, it has about 61.6% (for $E_r < 0$) chances to have a better design than the TPRD method. Thus, the presented method is more robust than the TPRD method, because the presented method considers model uncertainty while the TPRD method does not.

Example 5.2: Robust design for a low pass filter A low pass filter is shown in Figure 5.12. The design variable is the resistance R and the inductance L. The current I should be kept at a nominal value I_0 equal to $10A$. V and w are the amplitude and the frequency of the excitation voltage $v(t) = V \cos wt$, respectively. The current $i(t)$ is of the form $i(t) = I \cos(wt + \phi)$, with I and ϕ as the amplitude and the phase of the current. ■

The design variable d, the model parameter p, and the performance function Y are

$$d = p = \begin{bmatrix} R \\ L \end{bmatrix}, \quad Y = \begin{bmatrix} I \\ \phi \end{bmatrix}$$

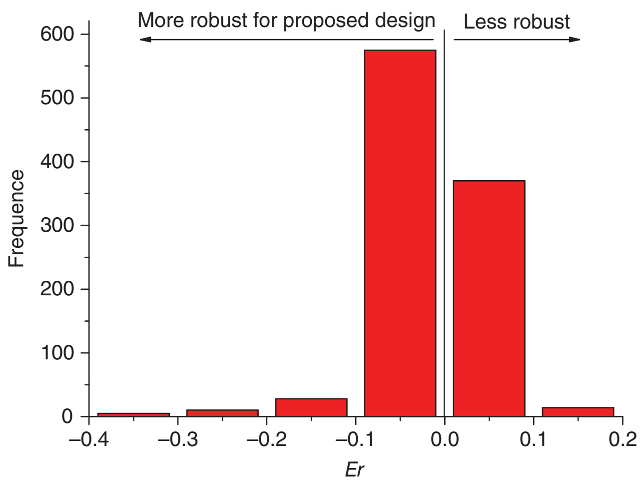


FIGURE 5.11 Comparison under parameter variations and model uncertainty

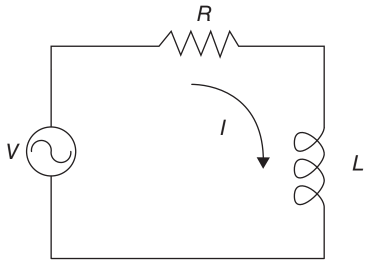


FIGURE 5.12 A low pass filter

The design variable d is the same with the design parameter p, which means that the design variable d has the variation around its nominal value. The design variable L_0 is decided by R_0 in order to make the nominal value $I_0 = 10A$.

The performance Y is expressed as

$$Y = \begin{bmatrix} \Delta Y_1 \\ \Delta Y_2 \end{bmatrix} = f(d,p) = \underbrace{\begin{bmatrix} \dfrac{V_0}{\sqrt{R^2 + w_0^2 L^2}} \\ \tan^{-1}\left(\dfrac{w_0 L}{R}\right) \end{bmatrix}}_{f_0} + \Delta f \tag{5.56}$$

where the model uncertainty $\Delta f = \begin{bmatrix} \frac{V}{\sqrt{R^2+w^2L^2}} - \frac{V_0}{\sqrt{R^2+w_0^2L^2}} \\ \tan^{-1}\left(\frac{wL}{R}\right) - \tan^{-1}\left(\frac{w_0L}{R}\right) \end{bmatrix}$ is unknown and a black box for designers.

The design variable d has variation around its design value and this variation follows Gaussian distribution with μ_d and σ_d. Moreover, the known nominal values V_0 and w_0 are 110 V and 60 Hz, respectively, which have variations $\Delta V = 0.8(L_0 + \Delta L)$ and $\Delta w = 100(R_0 + \Delta R)$ around their nominal values.

The nominal sensitivity matrix J_0 is formulated by

$$J_0 = \begin{bmatrix} \dfrac{\partial f_{0,1}}{\partial R} & \dfrac{\partial f_{0,1}}{\partial L} \\ \dfrac{\partial f_{0,2}}{\partial R} & \dfrac{\partial f_{0,2}}{\partial L} \end{bmatrix} \tag{5.57}$$

with $f_0 = \begin{bmatrix} f_{0,1} \\ f_{0,2} \end{bmatrix}$.

Then, the bound model (Equation 5.18) of the perturbation sensitivity matrix ΔJ is estimated from data with the confidence level $a = 99.73\%$. Given the nominal sensitivity matrix J_0 and the estimated model of the perturbation sensitivity matrix

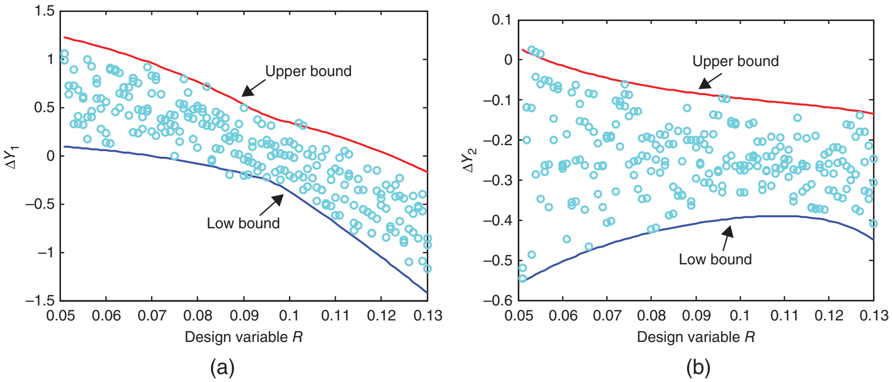


FIGURE 5.13 Bound modeling for the performance variation ΔY: (a) bound modeling for ΔY_1; (b) bound modeling of ΔY_2

ΔJ, the approximate ability of the model (Equation 5.17) to the practical system (Equation 5.44) is shown in Figure 5.13. Since almost all performance variations are limited within the bound, it has good ability to approximate the nonlinear system (Equation 5.56). It is clear that the bound modeling (Equation 5.18) is much easier than direct modeling of all data points (circle points in Figure 5.13), because the sampling data have both strong nonlinearity and random nature.

A robust design should be chosen from the design variable space $R_0 \in [0.05\Omega, 0.13\Omega]$ and $L_0 \in [7H, 11H]$ to minimize the performance covariance. The robust design variables d figured out by the TPRD method and the presented method are ($R_0 = 0.068\ \Omega$, $L_0 = 10.21$ H) and ($R_0 = 0.105\ \Omega$, $L_0 = 7.76$ H), respectively.

To demonstrate the effectiveness of the presented method, verification is carried out for comparison. Let ΔR and ΔL follow the normal distribution with mean $\mu_R = -0.05$, $\mu_L = -0.05$ and variance $\sigma_R = 0.01$, $\sigma_L = 0.01$. The variations ΔV and Δw are $0.8(L_0 + \Delta L)$ and $100(R_0 + \Delta R)$, respectively. A total of 1000 samples are taken for the comparison of the performance covariance with respect to both the parameter variations and the model uncertainty. From Table 5.2, we can see that the performance covariance gained by the presented method is smaller than that obtained by the TPRD method. As shown in Figure 5.14, it has about 86.4% (for $E_r < 0$) chances to have a better design than the TPRD method. Thus, the presented method is more robust than the TPRD method, because the presented method considers model uncertainty, while the TPRD method does not.

TABLE 5.2 Performance Comparison Under Parameter Variations and Model Uncertainty

$\|\sigma_Y\|_F$	TPRD Method	Presented Method
Covariance	0.044	0.019

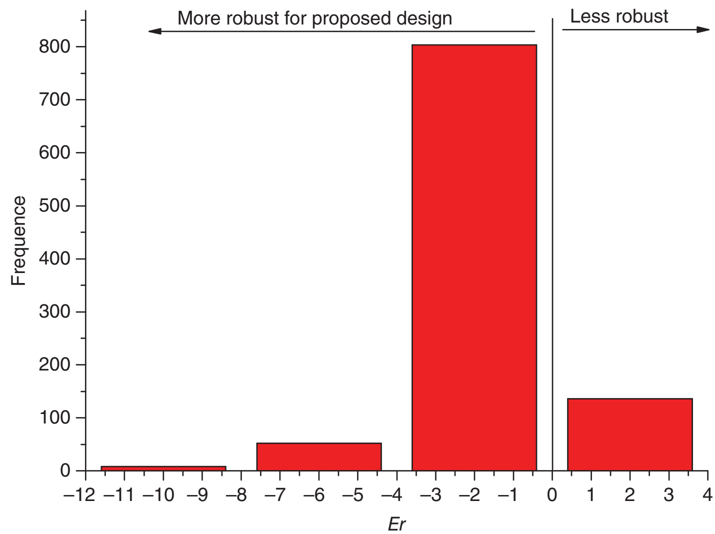


FIGURE 5.14 Comparison under parameter variations and model uncertainty

5.4.2 Deterministic Robust Design

Example 5.3: Structure uncertainty design Consider the structure uncertainty design problem:

$$Y = f + \Delta f \tag{5.58}$$

with $f = \begin{bmatrix} \ln(|\csc(d_1) - \tan(d_1)|) + 3d_2 \\ 3d_1 + d_2 \end{bmatrix}$

and $\Delta f = \begin{bmatrix} 0.05\ln(|\csc(d_1) - \tan(d_1)|) + \frac{0.1}{3}d_1^3 + 0.01d_2 \\ 0.01d_1 - 0.05\cos(d_2) \end{bmatrix}$ ■

From Equation 5.58, the performance variations ΔY can be expressed as

$$\Delta Y = J_0 \cdot \Delta d + \Delta J \cdot \Delta d \tag{5.59}$$

with $J_0 = \begin{bmatrix} \frac{1}{\sin(d_1)} & 3 \\ 3 & 1 \end{bmatrix}$, $\Delta J = \begin{bmatrix} \frac{0.05}{\sin(d_1)} + 0.1d_1^2 & 0.01 \\ 0.01 & 0.05\sin(d_1) \end{bmatrix}$, $\Delta d = \begin{bmatrix} \Delta d_1 \\ \Delta d_2 \end{bmatrix}$.

Here, the variation of the model parameter is replaced by the variation of the design variable. The matrix B_0, the singular value δ_i^0, and the condition number K can be calculated using Matlab program, respectively. The upper bound of $\|\Delta J\|_2$ is estimated from simulation data.

The design objective is to select the design variable d_1 from $d_1 \in [1, 2.5]$ to have a robust performance against uncertainty. Here, the nominal value d_2^0 of design variable

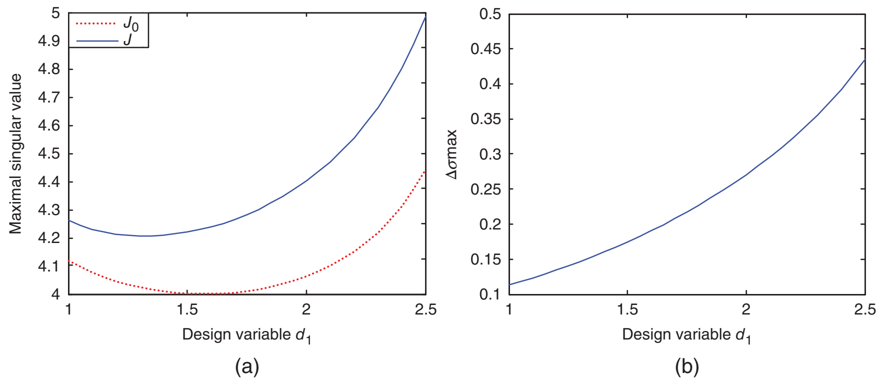


FIGURE 5.15 Maximal singular values of J_0 and J versus the design variable d_1: (a) $\delta_{\max}^0$ and $\delta_{\max}$; (b) the difference $\Delta\delta_{\max}$

d_2 is equal to 5. The maximal singular values of J_0 and J are shown in Figure 5.15. From Figure 5.15, it is clear that there exists the difference $\Delta\delta_{\max}$ between the maximal singular value $\delta_{\max}$ and $\delta_{\max}^0$ due to the effect of the model uncertainty.

From Figure 5.16, all $|\Delta\lambda|$ are smaller than the bound S_1, which is equal to $K \cdot (2\|J_0\|_2 + \|\Delta J\|_2) \cdot \|\Delta J\|_2$ in Equation 5.47. Moreover, the bound S_1 is minimal at d_1=1, which means the minimal variation $|\Delta\delta|$. From Figure 5.17, it is clear that the lower and upper bounds of $\delta_{\max}$ are very close at $d_1 = 1$. This also means that $\delta_{\max}$ is close to its nominal singular $\delta_{\max}^0$, which can be verified in Figure 5.15.

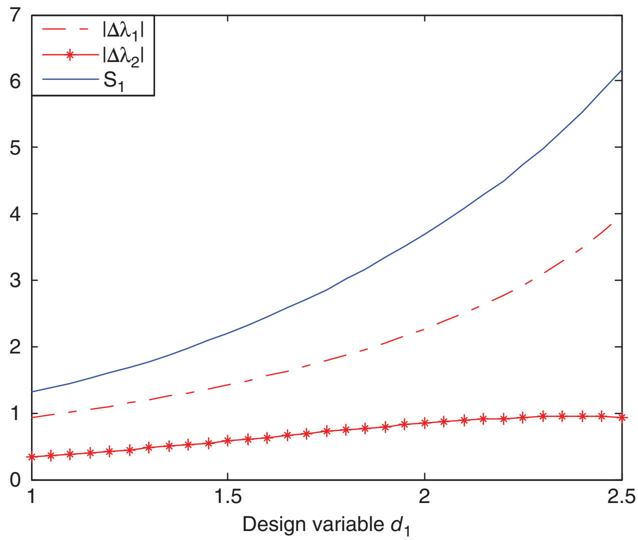


FIGURE 5.16 $|\Delta\lambda|$ and the bound S_1

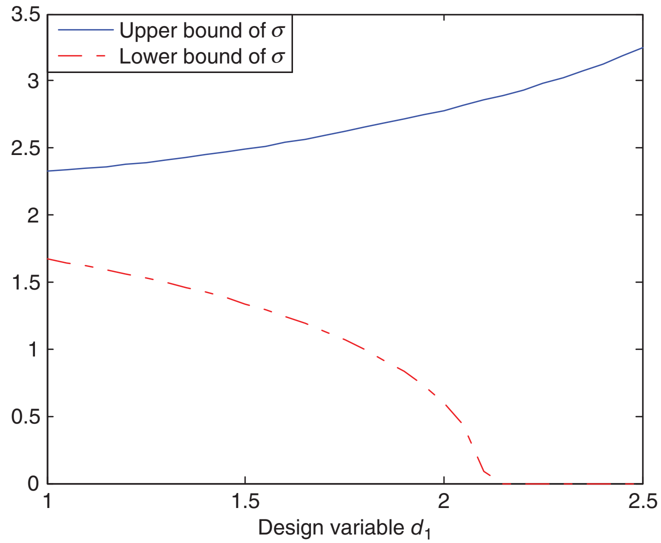


FIGURE 5.17 Lower and upper bounds of σ_{max}

Designs from different β are shown in Table 5.3. The traditional robust designs, which are figured out by Euclidean norm method or condition number method (Equation 5.14) (the results of these two methods are the same in this case), are taken as $\beta = 1$. From Equation 5.59, performance variations may be rewritten as

$$\|\Delta Y\|_2^2 = (r_1\Delta d_1 + 3.01\Delta d_2)^2 + (3.01\Delta d_1 + r_2\Delta d_2)^2 \tag{5.60}$$

with $r_1 = \frac{1.05}{\sin(d_1)} + 0.1d_1^2, r_2 = 1 + 0.05\sin(d_1)$.

For performance variations in Equation 5.60, smaller parameters r_1 and r_2 will have smaller performance variations, that is, better robustness. From Table 5.3, it is clear that both r_1 and r_2 obtained by the proposed robust design method with $\beta = 0.95$, 0.9, 0.85 are smaller than that by the traditional robust design methods (when $\beta = 1$). Thus the proposed robust design with $\beta = 0.95$, 0.9, 0.85 has better robustness than

TABLE 5.3 Robust Designs and Performance Under Different β

Weight β	1	0.95	0.9	0.85	0.8	0.75	0.7	0.5	0.2
Design d_1*	1.55	1.35	1.25	1.15	1.05	1	1	1	1
r_1	1.2905	1.2584	1.2627	1.2826	1.3207	1.3478	1.3478	1.3478	1.3478
r_2	1.05	1.0488	1.0474	1.0456	1.0434	1.0421	1.0421	1.0421	1.0421

TABLE 5.4 Performance Comparison Under Δd and the Model Uncertainty

Weight β		1	0.95	0.9	0.85	0.8	0.75
Design d_1*		1.55	1.35	1.25	1.15	1.05	1
$\|\Delta Y\|_2^2$	Mean	0.0182	0.0181	0.0181	0.0181	0.0182	0.0183
	Variance	2.7082×10^{-4}	2.6607×10^{-4}	2.6669×10^{-4}	2.6951×10^{-4}	2.6955×10^{-4}	2.7884×10^{-4}

the traditional robust designs. This is because the proposed robust design method considers model uncertainty, while the traditional robust design methods do not.

The above comparisons clearly show that a larger β should be selected since the model uncertainty Δf is smaller than 5% of the nominal model f. Thus, when the nominal model is dominant, a large β should be chosen.

To demonstrate the effectiveness of the proposed robust design method, verification is carried out by letting both Δd_1 and Δd_2 randomly vary in $[-0.05, 0.05]$. A total of 1000 samples are taken to compare the performance variation ΔY with respect to Δd under model uncertainty. From Table 5.4, we can see that the mean and variance of the performance variations ΔY gained by the proposed robust design method with $\beta = 0.95, 0.9, 0.85$ are smaller than the traditional design methods. It also shows that the best robust design is achieved when $\beta = 0.95$. The difference of the performance variations is defined as

$$T = \|\Delta Y_p\|_2^2 - \|\Delta Y_C\|_2^2 \tag{5.61}$$

where ΔY_p and ΔY_C are the performance variations gained by the proposed robust design method with $\beta = 0.95$ and the traditional robust design methods, respectively. The comparison in Figure 5.18 shows that the proposed robust design can have more than 63% (for $T < 0$) chance to have a better design than the traditional one. In other words, for every 100 designs, the new approach can get more than 63 better designs while the traditional one can get only less than 37 better designs. Only if this percentage is larger than 50%, then the proposed method is said to have better robustness than the traditional methods.

Example 5.4: Robust design of a damper The damper design example is shown in Figure 5.19. The design variables are mass M and damping coefficient C_d which are to be determined with the aim of keeping the magnitude of displacement X_0 at a nominal value of 3 m, while the magnitude F_0 of the excitation force $F(t) = F\cos(\omega \cdot t)$ and its pulsation ω undergo considerable variations. The displacement is equal to $X(t) = X\cos(\omega \cdot t + \phi)$, where ϕ is the phase. Moreover, the following relations exist: ■

$$X = \frac{F}{\omega\sqrt{C_d^2 + \omega^2 M^2}}, \quad \phi = \tan^{-1}\left(\frac{\omega M}{C_d}\right) \tag{5.62}$$

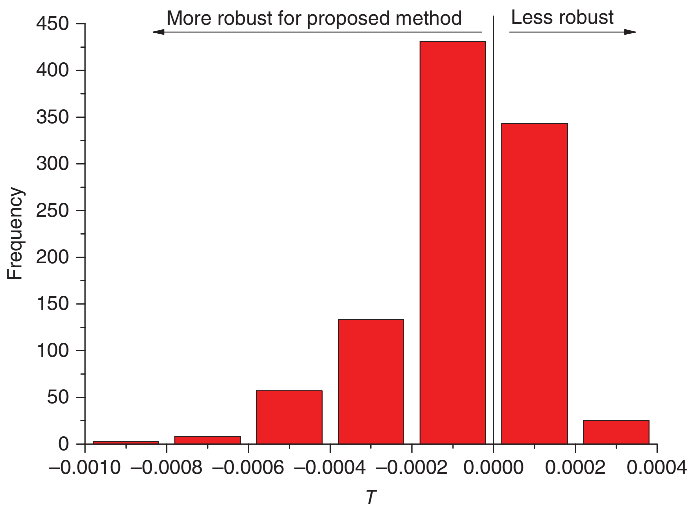


FIGURE 5.18 Comparison of the proposed robust design ($\beta = 0.95$) with the traditional robust design ($\beta = 1$)

The design variables d, the design parameters p, and the performance functions Y are

$$d = \begin{bmatrix} M \\ C_d \end{bmatrix}, \quad p = \begin{bmatrix} F \\ w \end{bmatrix}, \quad Y = \begin{bmatrix} X \\ \phi \end{bmatrix}$$

The design model with model uncertainty can be expressed as

$$\underbrace{\begin{bmatrix} X \\ \phi \end{bmatrix}}_{Y} = \underbrace{\begin{bmatrix} \dfrac{F_0}{\omega_0\sqrt{C_d^2 + \omega_0^2 M^2}} \\ \tan^{-1}\left(\dfrac{\omega_0 M}{C_d}\right) \end{bmatrix}}_{f(d)} + \underbrace{\begin{bmatrix} \Delta f_1 \\ \Delta f_2 \end{bmatrix}}_{\Delta f(d)} \tag{5.63}$$

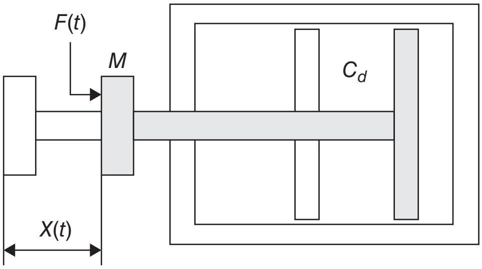


FIGURE 5.19 Damper

TABLE 5.5 Performance Comparison Under Δd, Δp, and Model Uncertainty

Weight β		1 (Euclidean Norm Method)	0.8	0.6	0.4	0.2	0	Condition Number Method
Design M^*		2.668	2.6560	2.6390	2.6060	2.5360	2.5220	3
Design $C_d{}^*$		39.9644	40.2829	40.7273	41.5686	43.2658	43.5919	25.059
$\|\Delta Y\|_2^2$	Mean	8.1287	8.1286	8.1286	8.1284	8.1284	8.1285	8.1594
	Variance	0.0076	0.0075	0.0074	0.0073	0.0071	0.0069	0.0154

Only the nominal model $f(d)$ is known to designers. The nominal value of F_0 and w_0 are 200 N and 20 rad/s, respectively, and the design parameters p have the variations $F = F_0 \pm 10$ and $w = w_0 \pm 2$. The objective is to select the design variables M from $M \in [2\text{kg}, 3\text{kg}]$ and C_d from $C_d \in [25\text{Ns/m}, 55\text{Ns/m}]$ to have a robust performance against both model uncertainty and parameter variations.

Designs from different β in Table 5.5 are compared with Euclidean norm method ($\beta = 1$) and the condition number method in Table 5.5. To demonstrate the effectiveness of the proposed robust design method, verification is carried out by letting ΔM, ΔC_d, ΔF, and Δw randomly vary in $(-0.1, 0.1)$, $(-2, 2)$, $(-10, 10)$, and $(-2, 2)$, respectively. A total of 1000 samples are taken to compare the performance variations ΔY with respect to Δd, Δp, and model uncertainty. From Table 5.5, we can see that the mean and variance of the performance variations ΔY gained by the proposed robust design method with $\beta = 0$, 0.2, 0.4 are smaller than the other two traditional design methods.

It is clear that a small β should be selected since the nominal model is not always dominant compared to the model uncertainty. For example, the nominal model $\phi_0 = \tan^{-1}(\frac{w_0 M}{C_d})$ and the model uncertainty $\Delta\phi = \tan^{-1}(\frac{wM}{C_d}) - \tan^{-1}(\frac{w_0 M}{C_d})$ are taken as 0.3323 and 0.4779, respectively, when w changes from the nominal value $w_0 = 20$ to $w = 18$, and $M = 2.5$ and $C_d = 40$. Thus, when model uncertainty has larger effect on system performances, a smaller β should be chosen.

In Figure 5.20, the proposed robust design with $\beta = 0.2$ is compared with the traditional robust design methods, including Euclidean norm method and the condition number method. It is clear that it has about 56.3% (for $T_1 < 0$) and 80.5% (for $T_2 < 0$) chances to have a better design than the traditional one. Only if this percentage is larger than 50%, then is better robustness achieved compared with the traditional methods. So the proposed robust design method is more robust than the other two traditional design methods, because the proposed robust design method considers model uncertainty.

5.5 SUMMARY

In this chapter, hybrid model/data-based deterministic and probabilistic robust design methods are presented to minimize the covariance of the partially unknown nonlinear

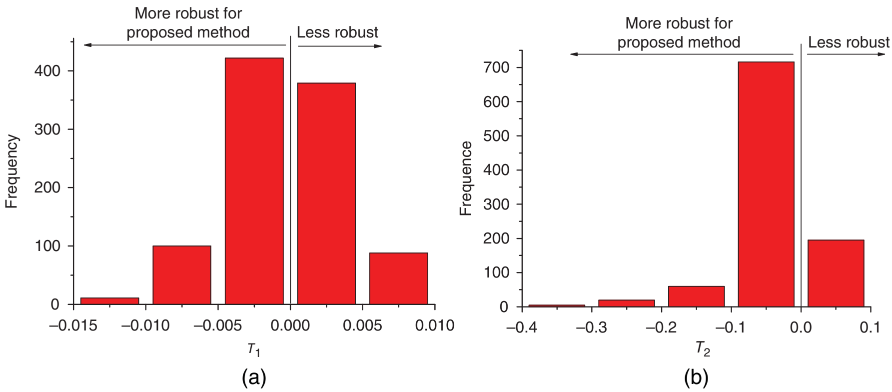


FIGURE 5.20 Comparison under Δd, Δp and model uncertainty: (a) Euclidean norm method versus proposed method; (b) condition number method versus proposed method

system. Bound modeling is easy to realize and can effectively estimate both the influence of model uncertainty and the nonlinearity of the nominal model. Since the presented methods properly integrate the data-based uncertainty compensation and the model-based robust design method, it can effectively design the system robustness even if both parameter variation and model uncertainty exist. These proposed methods are also compared with the traditional design methods on selected case studies and show their superiority when model uncertainty and parameter variation exist. These methods should be applicable in complex manufacturing environment.

PART III

ROBUST DESIGN FOR DYNAMIC SYSTEMS

CHAPTER 6

ROBUST EIGENVALUE DESIGN UNDER PARAMETER VARIATION—A LINEARIZATION APPROACH

System design is usually for static system. In this chapter, design for the dynamic system will be presented. The proposed approaches will integrate several methods, such as stability design, robust eigenvalue design, and tolerance design, to achieve the optimal dynamic performance, including both stability and robustness, under parameter variations.

6.1 INTRODUCTION

In manufacturing industry, many systems work in open loop without any external adjustment and control due to some physical and economic constraints. Operation performance fully depends on system design. For this kind of systems, a good design is crucial to achieve a desirable performance. Otherwise, the system will deviate from the ideal target. Most of the existing system methods are for static systems, a few for dynamic systems. The difficulty of dynamic system is that the system characteristic is varying over time. System stability is one of the major issues that is not considered in the static design.

The dynamic behavior of many systems in manufacturing is closely related to the eigenvalue of the Jacobian matrix that is often called the system or state matrix. The elements of the Jacobian matrix are dependent on the parameters that usually include design variables, operating environment variables, and manufacturing operations. Since variations of these parameters are unavoidable, the sensitivity of the eigenvalues

System Design and Control Integration for Advanced Manufacturing, First Edition. Han-Xiong Li and XinJiang Lu.

of this Jacobian matrix with respect to parameter variations is extremely important to the overall system performance (El-Kady and Al-Ohaly, 1997; Gürgöze, 1998; Orbak, Eskinat, and Turkay, 2004). If these eigenvalues are less sensitive to disturbances or parameter variations, the system will be robust. Under the closed-loop control, this robust performance is achieved by the robust eigenvalue assignment, which had been studied for decades (Ralph and Stephen, 1989; Kautsky, Nichols, and Dooren, 1985; Hu and Wang, 2002; Labibi, Marquez, and Chen, 2006). However, little work has been reported to address the robust eigenvalue design for the processes working under the open-loop environment.

In this chapter, two robust design methods are presented to integrate stability design and robust eigenvalue design for the dynamic process with weak nonlinearity.

- One is a linearization-based robust eigenvalue design proposed for small parameter variations. First, the stability theory is applied to obtain a set of design variables and bounds of their variations. The system will be stable when design variables stay within these bounds. Then, the robust eigenvalue design is developed to make the dynamic response less sensitive to variations. Furthermore, the tolerance space of the obtained robust design will be maximized to meet the specified performance requirement of the dynamic response.
- Another is a multi-model-based robust design proposed for large uncontrollable variations. A multi-model approach is initially developed to formulate the nonlinear relation between dynamic performance and model parameters. A stability design is then developed to guarantee the stability of the dynamic system under large uncontrollable variations. Moreover, a robust design is proposed to achieve dynamic robustness.

Finally, several examples will be provided to demonstrate and confirm the effectiveness of the proposed methods.

6.2 DYNAMIC DESIGN PROBLEM UNDER PARAMETER VARIATION

In this section, two problems on stability and robustness are concerned for design of the weak nonlinear dynamic system under parameter variation.

6.2.1 Stability Design Problem

Consider an autonomous dynamic system:

$$\dot{x} = f(x, d, p), \quad f(0, d, p) = 0 \tag{6.1}$$

where $x \in R^{\mathrm{n}}$ represents the state vector, $d = [d_1, \ldots, d_m]^T$ is design variables vector, which needs to be decided for the desirable performance, and p is the model parameter vector and has large parameter variation Δp around its nominal value p_0, $f(\cdot)$ is the nonlinear function vector.

Take Taylor series expansion of $f(x, d, p)$ at $x = 0$

$$\dot{x} = A(d,p)x + f_1(x,d,p) \tag{6.2}$$

where the Jacobian matrix $A(d,p) = \left.\frac{\partial f(x,\, d,\, p)}{\partial x}\right|_{x=0}$. If nonlinearity is weak, the system can be approximated into the linear system at the working point $x = x_0$, with the high-order term $f_1(x_0, d, p)$ ignored. For convenience, $A(d, p)$ is simply denoted as A.

Obviously, stability is critical to the dynamic system. According to the stability theory, the system is asymptotically stable if all eigenvalues of A have negative real parts (A is a Hurwitz matrix), and unstable if one or more eigenvalues of A have positive real parts. Thus, the objective of the stability design is to properly choose a set of the design variables d to make the real part of these eigenvalues $\lambda = [\lambda_1,\ldots, \lambda_n]$ of the Jacobian matrix A smaller than zero.

6.2.2 Dynamic Robust Design Problem

The relationship between an eigenvalue and its associated eigenvector can be expressed by

$$AU = UA \tag{6.3}$$

where $U = [u_1, \ldots, u_n]$ is the eigenvector matrix and u_i $(i = 1, \ldots, n)$ is the right eigenvector corresponding to the eigenvalueλ_i, $\wedge = diag[\lambda_1, \ldots, \lambda_n]$.

The transient response of the system state $x(t)$ can be expressed as

$$x(t) = Ue^{\wedge t}U^{-1}x(0) \tag{6.4}$$

where $x(0)$ is the initial value.

From Equation 6.4, the transient response of the system is characterized by the eigenvalues together with the eigenvectors. The eigenvalues determine the decay (or growth) rate of the response. The variation of eigenvalues will result in variation of the transient response. Thus, the eigenvalues should be selected to be insensitive to uncertainties as they are critical to system response. As illustrated in the specific constraint domain in Figure 6.1, eigenvalues should be constrained by both stability and robust performance. Under the guaranteed system stability, variations of the eigenvalues should be minimal with the smallest domain in Figure 6.1. Moreover, the tolerances of the design variables can be derived reversely from the constraint domain to guarantee the desired dynamic performance of the system.

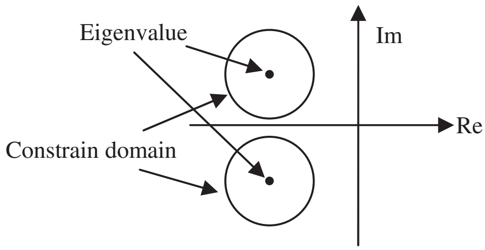


FIGURE 6.1 Eigenvalues constrained by both stability and robustness

6.3 LINEARIZATION-BASED ROBUST EIGENVALUE DESIGN

In this section, a linearization-based robust eigenvalue design is proposed for the system under small parameter variation. The objectives are

- to guarantee the system stability in the given parameter region;
- to achieve the dynamic robustness under parameter variation; and
- to obtain a best tolerance of the robust design variable.

For simplicity and without loss of generality, we assume that the design variables d have variations around their nominal values without considering parameter variations. Thus, matrix A is only the function of design variables d with variations Δd, rewritten as $A(d)$.

6.3.1 Stability Design

The objective of the stability design is to choose design variables d to have the real part of all eigenvalues $\lambda = [\lambda_1, \dots, \lambda_n]$ of the Jacobian matrix A smaller than zero. These design variables d can be figured out by solving the following feasible problem (P_1):

$$\begin{aligned} \textit{Feasible } (P_1)\text{: } & \textit{Find } (d) \\ & \textit{to make} \\ & \mathrm{Re}(\lambda_i(A(d))) < 0 \quad (i = 1, \dots, n) \\ & h(d) = 0 \\ & l(d) \leq 0 \end{aligned} \tag{6.5}$$

where $h(d)$ and $l(d)$ are constraints from other design aspects. All the possible solutions of the feasible problem (P_1) form a nominal stability parameter space S_n, in which every design variable will guarantee the system stability.

Generally, variations of design variables are inevitable. From the view of the robust stability, the maximum variation bound should be properly designed to allow larger tolerance for stability. When the design variables stay within these bounds, the system stability can still be maintained. The bigger these bounds are, the larger stability region the system will have. If these bounds are too small, it may cause a higher manufacture cost and the system may become unstable when encountering larger variations of the design variables. Thus, it is very important to have large variation bounds of design variables for the stability design. However, most stability designs are only to choose suitable design variables without consideration of their variation bounds at the design stage.

Thus, variation bounds of design variables around the nominal values should be figured out to restrain the variation effects of stability. First, the variation bounds of these design variables in the nominal stability parameter space S_n are figured out. Then, variations of the design variables are limited in the bounds $\{\Delta d_1 \in [-D_1, D_1], \ldots, \Delta d_m \in [-D_m, D_m]\}$ with $D_1, \ldots, D_m$ as the maximum values of $\Delta d_1, \ldots, \Delta d_\mathrm{m}$. When the design variables vary within the bounds, the system is still stable. Such bounds can be obtained from the solution of the following feasible problem (P_2):

$$\begin{aligned} \mathit{Feasible}\ (P_2): \quad & \mathit{Find}\ (D_1, \ldots, D_m \\ & \mathit{to\ make} \\ & \mathrm{Re}(\lambda_1(A(d + \Delta d))) < 0 \qquad (i = 1, \ldots, n) \\ & \Delta d_j \in [-D_j, D_j] \qquad (j = 1, \ldots, m) \\ & h(d + \Delta d) = 0 \\ & l(d + \Delta d) \le 0 \\ & d \in S_n \end{aligned} \tag{6.6}$$

Then, the design variables with the large variation bounds should be chosen from the nominal stability parameter space S_n. The reason is that the design variables with the large variation bounds have a good ability to restrain the variation effect of the design variables to stability. The larger these bounds are, the less sensitive (i.e., more robust) the design is. As an example, in Figure 6.2, design b is more robust than design a since the variation bound of design b is bigger. The triangles show the nominal value of the designs, and the dashed rectangles indicate the variation bounds in Figure 6.2.

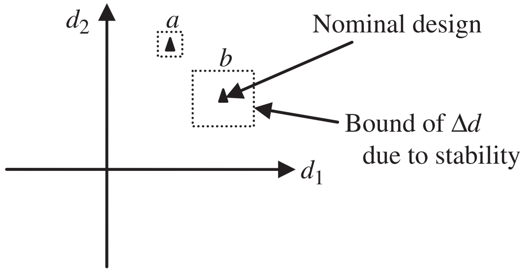


FIGURE 6.2 Variation bounds

Thus, it is important to judge whether the bounds $(D_1, \dots, D_m)$ are larger than the given level $(\varepsilon_1, \dots, \varepsilon_m)$. If the conditions $\varepsilon_1 \le D_1$ and … and $\varepsilon_{\mathrm{m}} \le D_{\mathrm{m}}$ are met, these stability design variables d can be accepted. All these accepted stability design variables d and their variation bounds form a stability parameter space S_{s}. When the design variables d vary within this stability parameter space S_{s}, the system stability will be maintained.

6.3.2 Robust Eigenvalue Design

In this section, the robust eigenvalue design will be chosen from the stability parameter space S_s to make variations of all eigenvalues minimal, so that the system dynamic response will be less sensitive to variations.

The eigenvalues λ_i $(i = 1, \dots, n)$ of the Jacobin matrix A, which are assumed to be distinct, are related to the corresponding eigenvector μ_i $(i = 1, \dots, n)$ of A by the equation

$$A\mu_i = \lambda_i \mu_i \tag{6.7}$$

According to the orthogonal theory of eigenvector, we have

$$\mu_i^T v_i = v_i^T \mu_i = \delta_{ij} \tag{6.8}$$

where δ_{ij} is Kronecher delta and v_{i} is left eigenvector with

$$A^T v_j = \lambda_j v_j \tag{6.9}$$

Differentiating Equation 6.7 with respect to the design variables d, we have

$$\frac{\partial A}{\partial d}\mu_i + A\frac{\partial \mu_i}{\partial d} = \frac{\partial \lambda_i}{\partial d}\mu_i + \lambda_i \frac{\partial \mu_i}{\partial d} \tag{6.10}$$

Multiplying Equation 6.10 by v_i^T and reordering the terms, we have

$$\frac{\partial \lambda_i}{\partial d} = v_i^T \frac{\partial A}{\partial d}\mu_i \tag{6.11}$$

If the design variables d have variations Δd at the operating point d_0, the system eigenvalue can be expressed as

$$\lambda_i(d_0 + \Delta d) = \lambda_i(d_0) + \left.\frac{\partial \lambda_i}{\partial d}\right|_{d_0} \Delta d + O(\Delta d) \tag{6.12}$$

where O(Δd) are high-order terms about Δd.

Inserting Equation 6.11 into Equation 6.12 and neglecting the high-order terms, a first-order model of the eigenvalue is obtained as

$$\lambda_i(d_0 + \Delta d) \approx \lambda_i(d_0) + \left. \left(v_i^T \frac{\partial A}{\partial d} \mu_i \right) \right|_{d_0} \Delta d \tag{6.13}$$

From Equation 6.13, the variation $\Delta\lambda_i$ of the eigenvalue with respect to variations of the design variables can be expressed as

$$\Delta\lambda = JX \tag{6.14}$$

with $\Delta\lambda_i = \lambda_i(d_0 + \Delta d_0) - \lambda_i(d_0)$, $\Delta\lambda = [\Delta\lambda_i \cdots \Delta\lambda_n]^T$,

$$J = \left| \begin{matrix} \left. \left(v_1^T \frac{\partial A}{\partial d_1} \mu_1 \right) \right|_{d_0} & \cdots & \left. \left(v_1^T \frac{\partial A}{\partial d_m} \mu_1 \right) \right|_{d_0} \\ \vdots & \ddots & \vdots \\ \left. \left(v_n^T \frac{\partial A}{\partial d_1} \mu_n \right) \right|_{d_0} & \cdots & \left. \left(v_n^T \frac{\partial A}{\partial d_m} \mu_n \right) \right|_{d_0} \end{matrix} \right|, \quad X = \begin{bmatrix} \Delta d_1 \\ \vdots \\ \Delta d_m \end{bmatrix}$$

The sensitivity matrix J of the eigenvalues is defined as the ratio of eigenvalue variations to parameter variations, which is very important to the system robustness (El-Kady and Al-Ohaly, 1997). Since J^TJ is usually a complex matrix, the real part and the imaginary part of the complex matrix J^TJ need to be considered separately. Therefore, Equation 6.14 may be rewritten as

$$\mathrm{Re}(\Delta\lambda) + i\mathrm{Im}(\Delta\lambda) = [\mathrm{Re}(J) + i\mathrm{Im}(J)]\, X \tag{6.15}$$

Easily we have

$$\mathrm{Re}(\Delta\lambda) = \mathrm{Re}(J)X \tag{6.16a}$$

$$\mathrm{Im}(\Delta\lambda) = \mathrm{Im}(J)X \tag{6.16b}$$

By taking a norm for both the real part and the imaginary part, we have performance limits for each part respectively.

$$(\mathrm{Re}(\Delta\lambda))^T \mathrm{Re}(\Delta\lambda) = X^T (\mathrm{Re}(J))^T \mathrm{Re}(J)X \tag{6.17a}$$

$$(\mathrm{Im}(\Delta\lambda))^T \mathrm{Im}(\Delta\lambda) = X^T (\mathrm{Im}(J))^T \mathrm{Im}(J)X \tag{6.17b}$$

The performance S of the system is defined and expressed as

$$S^2 = X^T BX \tag{6.18}$$

with $B = \mathrm{Re}(J)^T\mathrm{Re}(J) + \mathrm{Im}(J)^T\mathrm{Im}(J)$ and

$$S^2 = \|\mathrm{Re}(\Delta\lambda)\|_2^2 + \|\mathrm{Im}(\Delta\lambda)\|_2^2 = (\mathrm{Re}(\Delta\lambda))^T\mathrm{Re}(\Delta\lambda) + (\mathrm{Im}(\Delta\lambda))^T\mathrm{Im}(\Delta\lambda) \quad (6.19)$$

According to the singular value decomposition theory, the proper symmetric matrix B may be decomposed as

$$B = Vdiag(\sigma_1, \ldots, \sigma_m)V^T \quad (6.20)$$

where the symmetric matrix B describes the effect of the component variations to the system performance, σ_i is its singular value, and the corresponding orthogonal eigenvector is denoted as V_i, which is one element of $V = [V_1, \ldots, V_m]$.

Inserting Equations 6.19 and 6.20 into Equation 6.18, the performance will be directly related to the design variables as follows:

$$\|\mathrm{Re}(\Delta\lambda)\|_2^2 + \|\mathrm{Im}(\Delta\lambda)\|_2^2 = \sum_{i=1}^{m} \sigma_i z_i^2 \quad (6.21)$$

with $[z_1, \ldots, z_m]^T = V^T X$.

The performance in the m-dimensional space is a hyper-ellipsoid, as defined in Equation 6.21. Its two-dimensional (2D) projection is depicted in Figure 6.3, where both real and imaginary parts of eigenvalue are considered. It has similar characteristics presented by Zhu and Ting (2001) and Caro et al. (2005)

1. The performance S defined in Equation 6.19 is the same for every point on the hyper-ellipsoid.
2. The length of the ith principal axis of the hyper-ellipsoid is $(\|\mathrm{Re}(\Delta\lambda)\|_2^2 + \|\mathrm{Im}(\Delta\lambda)\|_2^2)/\sigma_i$. The smaller σ_i is, the longer the ith principal axis is. The longest principal axis and the shortest principal axis correspond to the least and most sensitive direction, respectively.

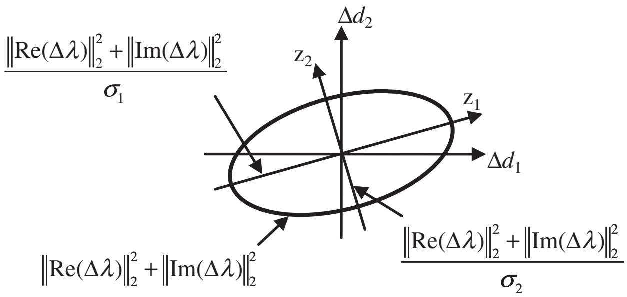


FIGURE 6.3 A 2D eigenvalue sensitivity ellipsoid

Thus, the robust design requires that all principal axes have larger length, especially the shortest principal axis. Since the shortest principal axis corresponds to the largest singular value $\sigma_{\max}$, if the largest singular value $\sigma_{\max}$ can be minimized, then the shortest principal axis will have relatively large length. So the design variables d for robust performance can be figured out from the following min-max optimization in the stability domain.

$$\begin{aligned}
&\min_{d}\ \max(\sigma_i) \\
&st. \\
&\|\mathrm{Re}(\Delta\lambda)\|_2^2 + \|\mathrm{Im}(\Delta\lambda)\|_2^2 = \sum_{i=1}^{m} \sigma_i y_i^2 \\
&d \in S_s
\end{aligned} \tag{6.22}$$

where $d \in S_s$ is the requirement of the system stability and feasibility. All possible solutions of the optimization (Equation 6.22) form a feasible parameter space S_f, on which the system stability and robustness are guaranteed.

6.3.3 Tolerance Design

In order to limit the variation of the dynamic response under the given performance constraint, tolerance in the feasible parameter space S_f should be figured out for the feasible design. If the performance constraint is specified as Y_r by users, then the performance S has to be limited in the constraint domain as follows,

$$S \leq Y_r \quad \text{or} \quad \left\|\mathrm{Re}(\Delta\lambda)\right\|_2^2 + \left\|\mathrm{Im}(\Delta\lambda)\right\|_2^2 \leq Y_r^2 \tag{6.23}$$

The tolerance design is to have the maximum variation space for design variables under the performance constraint Y_r and maintain the stability and robustness of the system. For simplicity, a 2D design problem will be discussed as shown in Figure 6.4. The variations of the design variables should be limited in the tolerance space S_t $\{\Delta d_1 \in [-\delta d_1, \delta d_1], \Delta d_2 \in [-\delta d_2, \delta d_2]\}$ with δd_1 and δd_2 as the largest variation of the

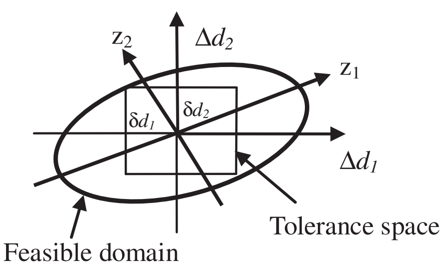


FIGURE 6.4 Tolerance synthesis

design variables d_1 and d_2. The tolerance space S_t is a rectangular space, as shown in Figure 6.4.

The principle of the tolerance design is to maximize the tolerance space within the sensitivity ellipsoid (feasible domain) (Caro, 2005). The tolerance of the design variables can be obtained from the following tolerance optimization:

$$\begin{aligned} \max_{\delta d_1, \delta d_2} \quad & \delta d_1 \delta d_2 \\ st. \quad & \\ & \|\mathrm{Re}(\Delta\lambda)\|_2^2 + \|\mathrm{Im}(\Delta\lambda)\|_2^2 \le Y_r^2 \\ & \delta d_1 \le D_1 \\ & \delta d_2 \le D2 \\ & d \in S_f \end{aligned} \tag{6.24}$$

where $\delta d_i \le D_i$ represents that tolerance is constrained in the stability parameter space S_s. This design will guarantee the system to have the desired stability and robustness when the design variables vary within the tolerance space.

6.3.4 Design Procedure

The presented robust design procedure is summarized as follows:

Step 1: Find the nominal stability parameter space S_n by solving the feasible problem (P_1). These nominal design variables guarantee the nominal system stability.

Step 2: Find the stability parameter space S_s within the nominal stability parameter space S_n by solving the feasible problem (P_2). When the design variables vary within this stability parameter space S_s, stability is still maintained.

Step 3: Find the robust eigenvalue design within the stability parameter space S_s by solving the optimization problem (Equation 6.22). This robust design minimizes the variations of eigenvalues so that the system dynamic response will be less sensitive to variations.

Step 4: Find the tolerance of the robust design by solving the optimization problem (Equation 6.24). If variations of the design variables are within this tolerance space, this design will be stable with the satisfactory dynamic performance.

6.4 MULTI-MODEL-BASED ROBUST DESIGN METHOD FOR STABILITY AND ROBUSTNESS

In this section, large uncontrollable variation will be considered. The system stability and dynamic robustness are nonlinearly influenced by the model parameters as the state matrix A is a nonlinear function of these parameters, as shown in Equation 6.2. A challenge is posed to find a suitable design variable d to guarantee the asymptotical stability as well as the satisfactory robust performance under large variation of uncontrollable parameters.

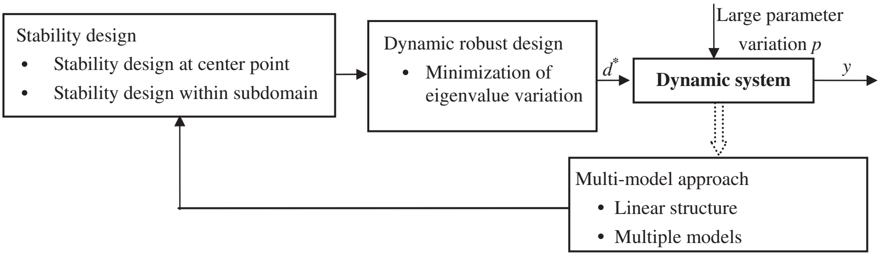


FIGURE 6.5 Multi-model-based robust design method

In order to meet this challenge, a new design approach is proposed as in Figure 6.5. The parameter space is first divided into many small subdomains. Through the idea of linearization, the complex nonlinear relation between dynamic performance and model parameter can always be decomposed into a series of linear relations at each subdomain. Modeling at each subdomain will be much easier than modeling the original nonlinear relation. Using the concept of multi-models, a stability design method is then developed to guarantee system stability under parameter variations. Afterwards, a dynamic robust design approach will be proposed to minimize the influence of parameter variation on the dynamic performance. Since this approach integrates the merits of both multi-models and robust design, it can effectively ensure stability and robustness of the system under large variation of uncontrollable parameters. This method has the following advantages:

- This modeling method considers the nonlinear influence of large parameter variation on the dynamic performance.
- The model built owns a well linear structure, which can be easily handled by existing robust design theories.
- Stability and dynamic robustness can be achieved by the proposed design method even under large variation of uncontrollable parameters.

6.4.1 Multi-Model Approach

It is well known that the system eigenvalues λ are critical to the dynamic system as they directly influence stability and robustness of the system. It is always a challenge to design the system eigenvalues as they are nonlinearly influenced by the model parameter with large uncontrollable variation. Here, a multi-model approach is first developed to handle this nonlinear influence. Although the approach is often employed in control field (Patil, 2012; Özkan, 2006; Chung, 2006), it has not yet been applied to system design for stability and robustness.

The basic concept of this multi-model method is illustrated in Figure 6.6 as an example of single parameter and single eigenvalue. The whole parameter variation space is divided into many subdomains at which a local linearization model is approximated around the center of the subdomain. Since every subdomain is small

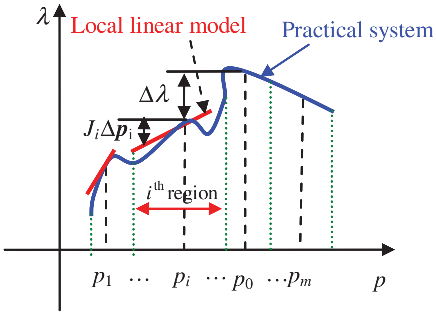


FIGURE 6.6 Multi-model approach

enough, each local linearization model can describe the behavior of the subdomain well. Thus, this modeling approach has the following advantages:

- It is a simple and straightforward way to decompose the complex modeling work (global) into a group of simple modeling tasks (local).
- It can represent any nonlinear relation well within a large parameter space.
- The model built has a well linear structure.

All these advantages will make the system easier to design by the existing design theory.

According to this modeling approach, the whole parameter variation space is first divided into m subdomains. Then, according to the orthogonal theory of eigenvector, the ith estimated eigenvalue $\lambda_{i,j}(i = 1, \dots, n)$ at the jth subdomain $(j = 1, \dots, m)$ is approximated by the below linear model

$$\lambda_{i,j}(d,p) \approx \lambda_{i,j}(d,p_j) + J_{i,j}(p - p_j) \tag{6.25}$$

with $J_{i,j} = (v_{i,j}^T \frac{\partial A}{\partial P} \mu_{i,j})\Big|_{p_j}$.
where p_j is the center of the jth subdomain, and $\mu_{i,j}$ and $v_{i,j}$ are the right eigenvector and the left eigenvector corresponding to the eigenvalue $\lambda_{i,j}(d, p_j)$, respectively. The derivation of Equation 6.25 may be found in Section 6.3.

Obviously, the number m of the subdomains depends on the system feature. If the nonlinear relation is strong and its model parameter has large variation, then m would be large or else it is small. Usually, the larger the m is, the better the modeling performance is, though it can produce a large computational cost. Therefore, there is always a compromise between modeling performance and computational cost when selecting m.

6.4.2 Stability Design

Stability design is to ensure the system to be stable under parameter variations by selecting a suitable design variable. Since the system eigenvalues under large variations can be decomposed into two parts in Equation 6.25, stability design can also be achieved by following two separate designs.

1. Nominal stability design (stability at the centre of every domain)—to deal the first item in Equation 6.25.

   It aims to find a nominal stability variable space S_n, which contains all designs that have negative eigenvalues $\lambda_{i,j}(d, p_j)$ of the nominal matrix $A(d, p_j)$ at center of every subdomain.
2. Stability design within subdomain—to deal with the second item in Equation 6.25.

   It aims to choose stability design variable from S_n to ensure the system stability within every subdomain in spite of uncontrollable parameter variations.

The above two designs can ensure the system to be stable even under large uncontrollable parameter variations.

6.4.2.1 Stability Design at Center Point This design is to make the real part of all eigenvalues $\lambda_{i,j}(d, p_j)$ of the nominal matrix $A(d, p_j)$ to be negative by selecting suitable design variables. Such design variables d can be figured out by the following feasible problem (P_3):

$$\begin{aligned} \textit{Feasible } (P_3): \quad & \textit{Find } (d) \\ & \textit{to make} \\ & \mathrm{Re}(\lambda_{i,j}(d, p_j)) < 0 \quad (i = 1, \ldots, n; j = 1, \ldots, m) \\ & h(d) = 0, l(d) \leq 0 \end{aligned} \tag{6.26}$$

where $\mathrm{Re}(\lambda)$ denotes the real part of.λ All possible solutions of the feasible problem (P_3) form a nominal stability variable space S_n, in which any design variable will ensure system stability at center points of all subdomains.

6.4.2.2 Stability Design Within Subdomain Then, the real part of the eigenvalue $\lambda_{i,j}(d, p)$ at every subdomain should be negative. The variation $\Delta p_{i,j}$ of the ith parameter at the jth subdomain is less than its own bound value $S_{i,j}$, namely the width of its corresponding subdomain. Thus, the parameter variation $P - P_j$ is less than its corresponding bound value $S_j = [S_{1,j}, \ldots, S_{n,j}]$. To ensure stability at every subdomain, the stability theorem is proposed below.

Theorem: Consider this nonlinear design problem (Equation 6.1) under parameter uncertainty. Given the nominal stability variable space S_n, if $\sum_{k=1}^{n} |J_{k,ij} S_{k,ij}|$ is smaller than $|\mathrm{Re}(\lambda_{i,j}(d, p_j))|$, where k refers to the kth element of its corresponding vector, then all eigenvalues of $A(d, p)$ have negative real parts and the system (Equation 6.1) is asymptotically stable even if parameter variations exist.

Proof: Since all design variables are within the nominal stability variable space S_n, $\mathrm{Re}(\lambda_{i,j}(d, p_j))$ is less than zero. Since S_j is the upper bound of the parameter variation $P - P_j$ at the jth subdomain, according to the matrix inequality, it has

$$|J_{ij}(p - p_j)| \leq \sum_{k=1}^{n} |J_{k,ij} S_{k,ij}| \tag{6.27}$$

■

From the inequality (Equation 6.27), if $\sum_{k=1}^{n} |J_{k,ij} S_{k,ij}|$ is smaller than $|\mathrm{Re}(\lambda_{i,j}(d, p_j))|$, then

$$|J_{ij}(p - p_j)| < |\mathrm{Re}(\lambda_{i,j}(d, p_j))| \tag{6.28}$$

Thus, from Equation 6.25, it is clear that the real part of $\lambda_{i,j}(d, p)$ is less than zero if $\sum_{k=1}^{n} |J_{k,ij} S_{k,ij}|$ is less than $|\mathrm{Re}(\lambda_{i,j}(d, p_j))|$. This means that the system (Equation 6.1) is asymptotically stable.

According to this theorem, the stability design can be figured out by solving the following feasible problem (P_4):

$$\begin{aligned} \textit{Feasible } (P_4): \quad & \textit{Find } (d) \\ & \textit{to make} \\ & \sum_{k=1}^{n} |J_{k,ij} S_{k,ij}| < |\mathrm{Re}(\lambda_{i,j}(d, p_j))| \quad (i = 1, \ldots, n;\ j = 1, \ldots, m) \\ & d \in S_n \end{aligned} \tag{6.29}$$

The inequalities $\sum_{k=1}^{n} |J_{k,ij} S_{k,ij}| < |\mathrm{Re}(\lambda_{i,j}(d, p_j))|$ can guarantee all system eigenvalues to be negative even under large uncontrollable parameter variation. All possible solutions of the feasible problem (P_4) form a stability variable space S_s, in which any design variable will guarantee the system stability even under large uncontrollable parameter variations.

Feasible problems (Equations 6.26 and 6.29) widely exist in the system design/control and can be solved by common inequality solver methods, such as discretization algorithm (Blanco and Bandoni, 2003; Dimitriadis and Pistikopoulos, 1995).

6.4.3 Dynamic Robust Design

This design is developed to achieve the dynamic robustness by making the system eigenvalues less sensitive to uncertainty. If the system (Equation 6.1) is robust, parameter variation will have less influence on the system eigenvalues. This means that the eigenvalues of all subdomain models are close to its nominal eigenvalues.

The nominal eigenvalues $\lambda(d, p_0)$ can be calculated from the following nominal model under $p = p_0$

$$\dot{x} = A(d, p_0)x \tag{6.30}$$

Define

$$\Delta\lambda_{i,j} = \lambda_{i,j}(d, p) - \lambda_i(d, p_0)$$

and

$$\Delta\lambda_{i,j}^0 = \lambda_{i,j}(d, p_j) - \lambda_i(d, p_0) \tag{6.31}$$

From Equations 6.25 and 6.31, we have

$$\Delta\lambda_{i,j} = \Delta\lambda_{i,j}^0 + J_{i,j}(P - P_j) \tag{6.32}$$

From Equation 6.32, it is clear that system eigenvalues are dependent on two parts: the center variation $\Delta\lambda_{i,j}^0$ and the domain variation $J_{i,j}(p - p_j)$. The center variation represents the distance between the nominal value p_0 and the center of every domain, and the domain variation represents performance variation within each domain around its center.

According to the matrix norm theory, Equation 6.32 may be rewritten as

$$\begin{aligned} |\Delta\lambda_{i,j}| &= \left|\Delta\lambda_{i,j}^0 + J_{i,j}(p - p_j)\right| \\ &\leq \left|\Delta\lambda_{i,j}^0\right| + \sum_{k=1}^{n} |J_{k,ij} S_{k,ij}| \end{aligned} \tag{6.33}$$

If $|\Delta\lambda_{i,j}^0| + \sum_{k=1}^{n} |J_{k,ij} S_{k,ij}|$ is very small, then

1. $\lambda_{i,j}(d, p)$ is close to $\lambda_i(d, p_0)$; and
2. $\lambda_{i,j}(d, p)$ is less sensitive to parameter variation p.

Under this sense, if all $|\Delta\lambda_{i,j}^0| + \sum_{k=1}^{n} |J_{k,ij} S_{k,ij}|$ in all subdomains are very small, then $\lambda(p, d)$ will be close to $\lambda(p_0, d)$ and, thus, this dynamic system is robust. Thus, the design variable d for the robust eigenvalue design can be figured out by solving the following robust design problem

$$\begin{aligned} &\min_{d} \sum_{i=1}^{n} \sum_{j=1}^{m} \left(\left|\Delta\lambda_{i,j}^0\right| + \sum_{k=1}^{n} \left|J_{k,ij} S_{k,ij}\right| \right) \\ &s.t. \quad d \in S_s \end{aligned} \tag{6.34}$$

where $d \in S_s$ is the requirement of stability and feasibility. This min-max optimization problem (Equation 6.34) is a common optimization problem in the system design/control. The solution can be easily obtained using the traditional gradient algorithm if their objective functions are convex function, and difficulties could be encountered if they are nonconvex and nonlinear. Then, some intelligent algorithms, such as particle swarm optimization and genetic algorithm, could be very useful to explore the possible solution. The solution of this robust design problem (Equation 6.34) can ensure the stability as well as the dynamic robustness of the system under large uncontrollable parameter variations.

6.4.4 Summary

Many manufacturing systems are often nonlinearly influenced by model parameters with large uncontrollable variations. The proposed design will be able to ensure system stability as well as system robustness under uncontrollable variations. Its design procedure is summarized as follows:

Step 1: This nonlinear relation between the dynamic performance and model parameter is modeled using the multi-model method. Since every subdomain is made small enough, local linearization model could approximate each subdomain properly. Modeling at each subdomain will be much easier than modeling of the original nonlinear relation. The model built also has a well-designed linear structure.

Step 2: The nominal stability parameter space S_n is found by solving the feasible problem (P_3). These nominal design variables could ensure system stability at all center points of the subdomains.

Step 3: The stability parameter space S_s is found within the nominal stability parameter space S_n by solving the feasible problem (P_4). These stability design variables could ensure system stability under large uncontrollable parameter variations.

Step 4: The robust dynamic design is found within the stability parameter space S_s by solving the robust design problem (Equation 6.34). This dynamic robust design could effectively achieve the desired dynamic response under large uncontrollable variations.

Obviously, when there is only one subdomain, the above proposed design approach will be the same, with the linearization-based robust eigenvalue design method presented in Section 6.3. Thus, that previous approach in Section 6.3 can be considered as a special case of the newly proposed approach in this section.

6.5 CASE STUDIES

6.5.1 Linearization-Based Robust Eigenvalue Design

Consider the laval rotor systems in Example 1.3 given in Section 1.1.2 of Chapter 1. In this design, we assume that model uncertainty in the example may be neglected and

TABLE 6.1 Parameter Values

d_i	d_e	d_b	K	K_b	Ω
1 (N•s/m)	5 (N•s/m)	10 (N•s/m)	100 (N/m)	400 (N/m)	50 Hz

only variations of design variables are considered. Thus, the state-space equations for the rotor system are

$$\dot{x} = Ax \tag{6.35}$$

with $A = A_0 = \begin{bmatrix} -M^{-1}D & -M^{-1}R \\ I_{4\times4} & 0_{4\times4} \end{bmatrix}, x \begin{bmatrix} \dot{q} \\ q \end{bmatrix}$.

The parameter values are shown in Table 6.1.

The objective is to select the design variables m and m_b from $m \in$ [1kg, 4kg] and $m_b \in$ [1kg, 5kg] to make the system stable and minimize the eigenvalue sensitivity.

6.5.1.1 Design of the Nominal Stability Parameter Space S_n

A nominal stability parameter space S_n is figured out by solving the feasible problem (P_1). This space S_n is shown in Figure 6.7, where the blank space stands for the unstable design and the shadow space for the stable design.

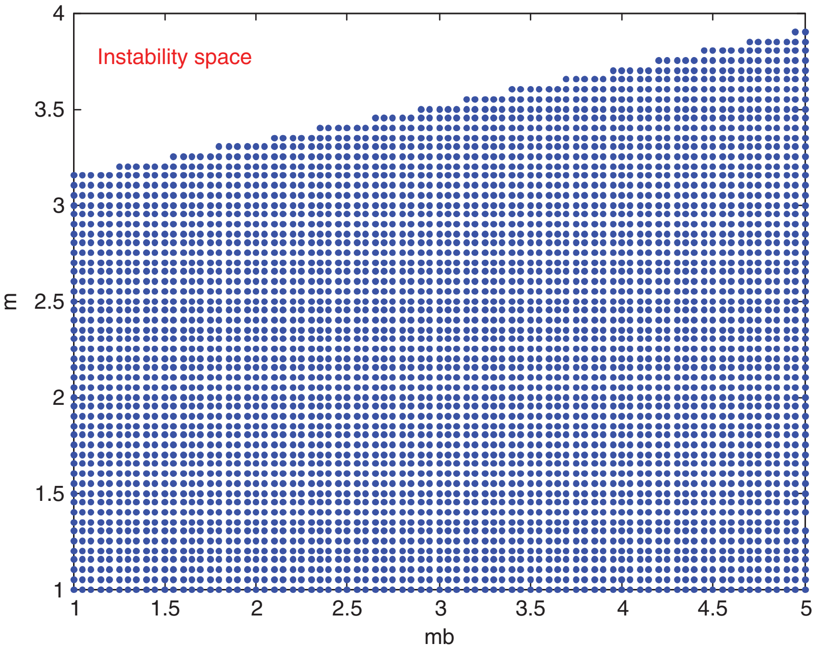


FIGURE 6.7 Nominal stability parameter space S_n

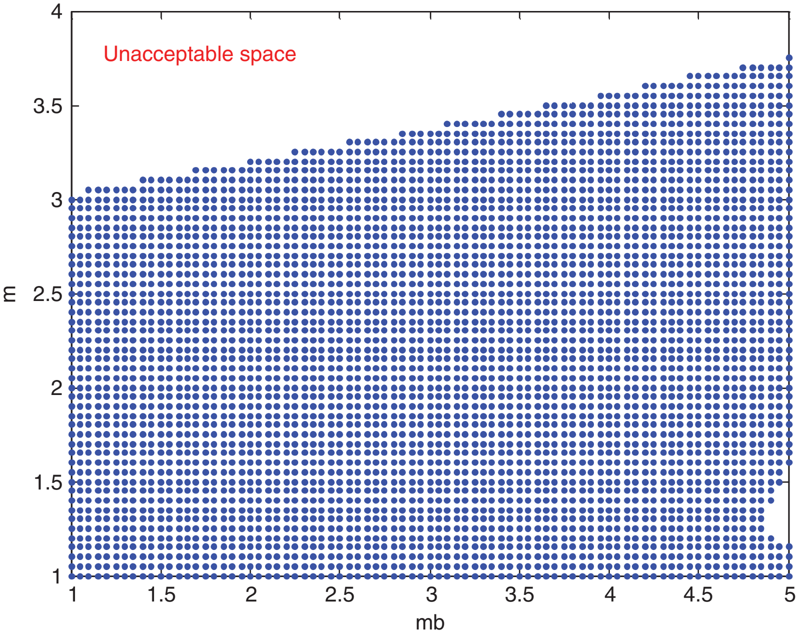


FIGURE 6.8 Stability parameter space S_s

6.5.1.2 Design of the Stability Parameter Space S_s Then, the stability parameter space S_s is figured out by solving the feasible problem (P_2) from a nominal stability parameter space S_n, with the given level $\varepsilon_1 = \varepsilon_2 = 0.2$. This space S_s is shown in Figure 6.8, where the blank spaces stand for the failure design space and the shadow space stands for the acceptable design space.

6.5.1.3 Robust Eigenvalue Design The eigenvalues of the system with the nominal mass $m = 2$ kg and $m_b = 2$ kg are obtained from matrix A in Equation 6.35. The nominal mass m and m_b have small variations $\Delta m = 0.02$ kg and $\Delta m_b = 0.02$ kg, respectively. The exact eigenvalue λ_{real} of the system with mass $m + \Delta m$ and $m_b + \Delta m_b$ is calculated from matrix A in Equation 6.35 via MATLAB and the eigenvalue λ_{model} is estimated approximately from its first-order model given in Equation 6.13. The relative approximation error η is defined as below

$$\eta = \frac{|\lambda_{real} - \lambda_{model}|}{|\lambda_{real}|} \tag{6.36}$$

The exact eigenvalues, their approximations, and the relative approximate error given in Table 6.2 demonstrate the effectiveness of the simple first-order model approximation.

TABLE 6.2 Estimated Eigenvalues and Their True Value as $m = 2$ kg and $m_b = 2$ kg

Eigenvalues	Δm	Δm_b	λ_1, λ_2	$\eta_{1,2}$	λ_3, λ_4	$\eta_{3,4}$	λ_5, λ_6	$\eta_{5,6}$	λ_7, λ_8	$\eta_{7,8}$
True value	0.02 kg	0.02 kg	−0.35 ± 6.27i	0.5%	−2.50 ± 6.24i	0.54%	−3.92 ± 15.85i	0.41%	−1.65 ± 15.82i	0.44%
Estimated value			−0.36 ± 6.30i		−2.52 ± 6.27i		−3.95 ± 15.92i		−1.66 ± 15.89i	

The robust parameters and the min-max singular value gained from Equation 6.22 are shown in Table 6.3, where we can see that the real parts of all eigenvalues are negative. So the design not only achieves robustness at $m = 2$ kg and $m_b = 4.6$ kg but also guarantees the system to be stable.

6.5.1.4 Tolerance Design

Assume that the performance constraint is defined as $Y_r = 3.1 \times 10^{-16}$. The largest tolerance space of the selected robust design variables $m = 2$ kg and $m_b = 4.6$ kg will be solved from Equation 6.24. The tolerance space and the maximum variation of the design variables are shown in Table 6.4. These tolerances are easy to realize because these tolerances 0.33 and 0.2 kg are easy to manufacture and measure using a common manufacture process and a common balance.

6.5.1.5 Design Verification

The performance S^2 as defined in Equation 6.18 due to the variations Δm and Δm_b is

$$S^2 = \|\mathrm{Re}(\Delta\lambda)\|_2^2 + \|\mathrm{Im}(\Delta\lambda)\|_2^2$$

The difference $e(m_i, m_{b,j})$ between the performance variation $S^2_{(m_i, m_{b,j})}$ obtained in the design variables ($m = m_i$, $m_b = m_{b,j}$) and S^2_{robust} obtained in the robust design ($m = 2$ kg, $m_b = 4.6$ kg) is defined as

$$e(m_i, m_{b,j}) = S^2_{(m_i, m_{b,j})} - S^2_{robust}$$

Let Δm and Δm_b vary randomly in [−0.01, 0.01]. A total of 1000 samples are taken to check the robust design. From Figure 6.9, we can see that the mean and variance of all $e(m_i, m_{b,j})$ are larger than zero, which shows that the robust design has smaller performance variations than other designs and is thus more robust than other designs.

TABLE 6.3 Stability-Based Robust Parameters

m	m_b	λ_1, λ_2	λ_3, λ_4	λ_5, λ_6	λ_7, λ_8	Min–Max Singular Value
2 kg	4.6 kg	−0.20 ± 11.02i	−2.57 ± 10.85i	−0.49 ± 6.02i	−2.13 ± 6.19i	1.0048×10^{-15}

TABLE 6.4 Performance-Based Tolerance

Y_r	Largest Tolerance Space	Tolerance δm	Tolerance δm_b
3.1×10^{-16}	0.066	0.33	0.2

6.5.2 Multi-Model-Based Robust Design Method

Example 6.1 Consider the following nonlinear design problem.

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0.094864(p^2 - 2) - 1 & 1.0094864 \\ 0.05d & -\sin(p) - 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \tag{6.37}$$

There is uncontrollable variation Δp around its nominal value $p_0 = 3.25$ due to manufacturing error and operating error. The design variable d can be chosen from its design space $d \in [-7, 2]$ to stabilize the system and achieve the desired dynamic robustness under uncontrollable variation Δp. ■

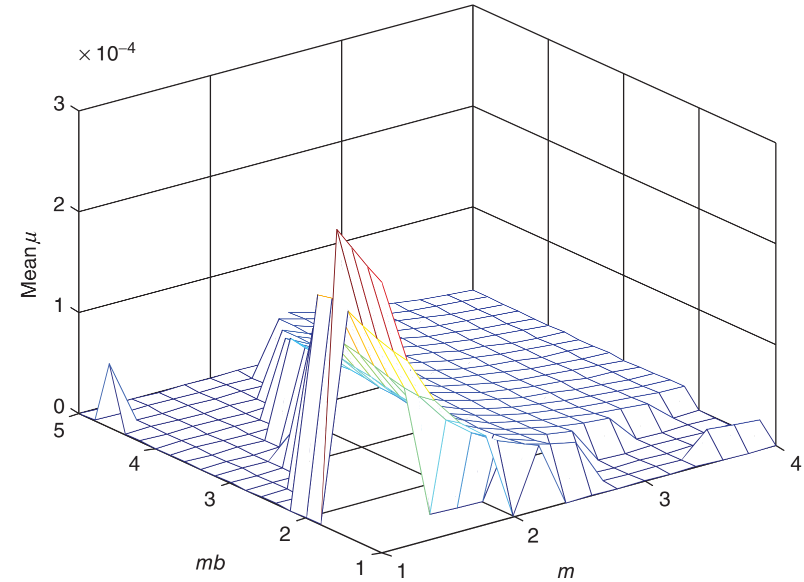
6.5.2.1 Stability Design Stability domains, obtained respectively by the proposed stability design method and the linearization-based stability design method (LSDM) presented in Section 6.3, are shown in Figure 6.10. It is easily seen that the stability domain of the proposed method is [−7, 1.2] and that of LSDM is [−7, 2]. The proposed design shows an unstable region at [1.2, 2], while the LSDM does not. A further analysis, as shown in Table 6.5, has confirmed that this unstable region does exist as the corresponding eigenvalues are positive. Thus, the proposed design method is demonstrated to be more effective than the existing method.

6.5.2.2 Robust Design The effectiveness of the proposed robust design method is verified when Δp randomly varies in $[-0.05\mathrm{p}_0, 0.05\mathrm{p}_0]$. The parameter variation domain is divided into five subdomains for the multi-model approach. The robust solution is $d = -7$. A total of 1000 samples are taken to compare variation of the transient response with respect to Δp.

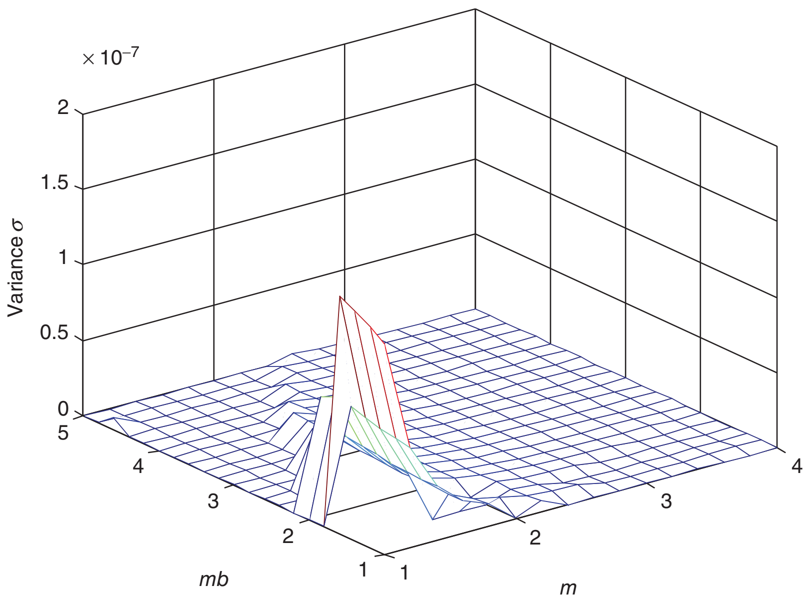
Performance variations under the constant initial condition and Δp are shown in Figure 6.11. It is clear that the transient responses of the state x_1 and x_2 are stable and robust under parameter variation.

Moreover, let the initial value randomly vary in [1, 1.2]. From Figure 6.12, it is clear that transient responses of state x_1 and x_2 are also stable and robust under parameter variation.

6.5.2.3 Design Comparison Effectiveness of the proposed robust design method is demonstrated in comparison with the linearization-based robust eigenvalue design method (LREDM) when Δp randomly varies in $[-0.05\mathrm{p}_0, 0.05\mathrm{p}_0]$. The performance comparison is shown in Table 6.6, where the transient response of the



(a)



(b)

FIGURE 6.9 Mean and variance of the difference $e(m_i, m_{b,j})$ (a) mean; (b) variance

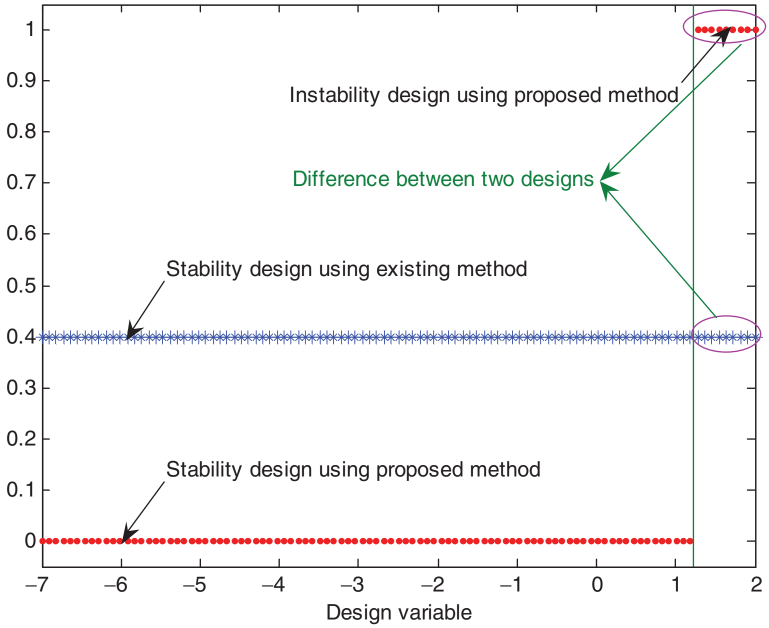


FIGURE 6.10 Stability design

TABLE 6.5 Instability Design

D	p	λ_1	λ_2
1.28	3.409	0.00063	−0.82
1.55	3.396	0.0079	−0.851
1.82	3.38	0.0123	−0.882
2	3.36	0.0112	−0.907

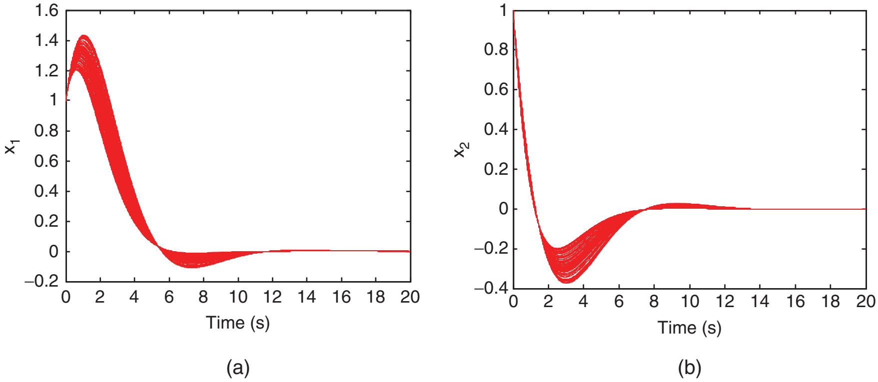


FIGURE 6.11 Performance under parameter variation Δp: (a) transient response of $x_1(t)$; (b) transient response of $x_2(t)$

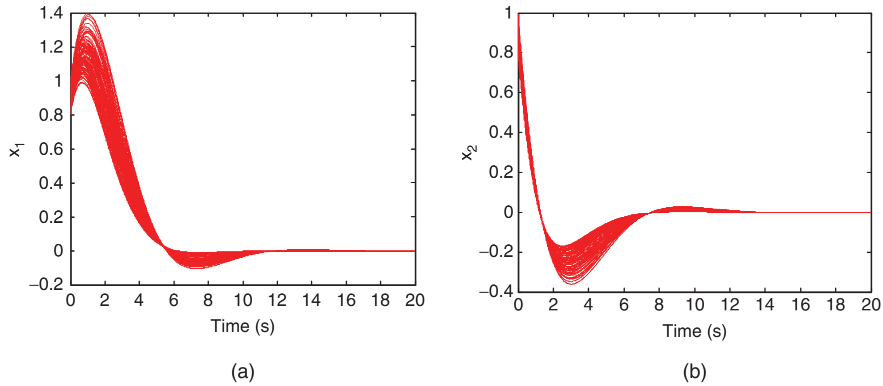


FIGURE 6.12 Performance under variation of initial condition and Δp: (a) transient response of $x_1(t)$; (b) transient response of $x_2(t)$

existing design ($d = -2.5$) clearly has a larger performance variation than the proposed method under both constant and variable initial values. Thus, the proposed method is less sensitive to parameter variations than the existing method.

Example 6.2 The actuating process is very important in providing a desired displacement in the manufacturing industry. A key requirement for a high-quality actuator is to provide a dynamic response that is less sensitive to parameter variations, thus making its dynamic behavior easy to estimate and predict. ■

As shown in Figure 6.13, a practical high-precision magnetic actuator consists of a pair of linked and guided rigid masses m_1 and m_2. The magnetic force is used to provide the required actuating force, which is produced by the electromagnetic induction through the current i on an iron core. It therefore has advantage in obtaining the actuating force without any contact. Due to cost minimization from users, the constant power should be set without any adjustment during the operation. Equivalently, this system works in the open loop (without external control).

TABLE 6.6 Performance Comparison Under Different Design Methods

	Performance variation: $\Delta = \sqrt{\|x_1(t) - x_{10}(t)\| + \|x_2(t) - x_{20}(t)\|}$ (where x_{10}, x_{20} are responses under nominal model parameter)			
	When the initial value is constant		When the initial value is variable	
	Mean	Variance	Mean	Variance
LREDM	103	5241	105	3772
Proposed method	47.29	1080	48.46	771

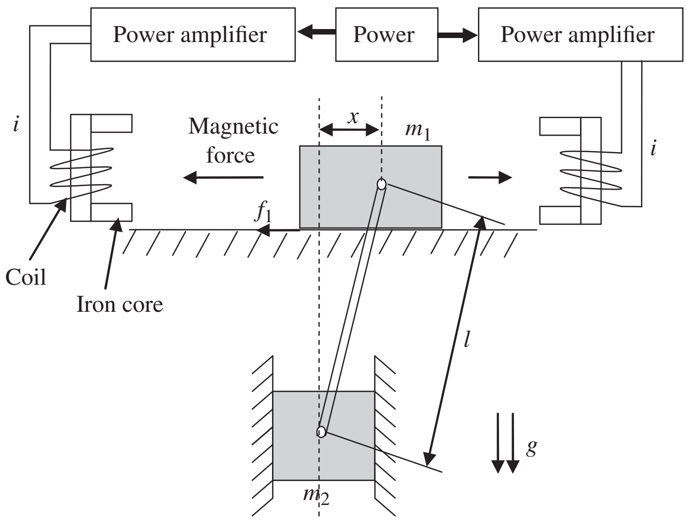


FIGURE 6.13 Diagram of a practical vibration machine

The magnetic force f and the resistance f_1 may be expressed by

$$f = \gamma \mu_0 N^2 i^2 A x \tag{6.38a}$$

$$f_1 = cx \tag{6.38b}$$

where i is the current of coil, N is the turn number of coil, μ_0 is the magnetic conductivity of air, x is the displacement of $m_{1,}$ A is the area of magnetic pole; and c is coefficient; γ—its nominal value γ_0 is known but there is unknown variation around its nominal value γ_0.

From Equation 6.38a, this magnetic force may be equivalent to a spring system and its stiffness k is equal to

$$k = \gamma \mu_0 N^2 i^2 A \tag{6.39}$$

According to Newton's second law and conservation of momentum, the motions of m_1 are derived at the working point ($x_{10} = 5$ and $x_{20} = 6$) as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \dfrac{(-K - m_2 g + c)\sqrt{l^2 - x_{10}^2}}{m_1(l^2 - x_{10}^2) + m_2 x_{10}^2} & \dfrac{-m_2 l^2 x_{10} x_{20}}{m_1(l^2 - x_{10}^2) + m_2 x_{10}^2 (l^2 - x_{10}^2)} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{6.40}$$

where $x_1 = x$ and $x_2 = \dot{x}$.

In this magnetic actuator design, the design variable d includes k and l, with nominal values chosen from their design space $k \in [7, 15]$ and $l \in [6, 11]$. However,

TABLE 6.7 Instability Design

D	λ_1	λ_2
$k = 8.21, l = 6$	0.003	−0.859
$k = 8.18, l = 8$	0.0062	−0.862
$k = 8.19, l = 10$	0.0088	−0.465

there are uncontrollable variations Δk and Δl around their nominal values. These variations could come from the magnetic leak and the manufacturing error. A key issue for the high-precision actuator is to choose suitable design variables to ensure both stability and robustness under uncontrollable uncertainty.

6.5.2.4 Stability Design The stability domains obtained by the proposed stability design method and the LSDM are $\{(k, l) | 8.7 \leq k \leq 15, 6 \leq l \leq 11\}$ and $\{(k, l) | 8.3 \leq k \leq 15, 6 \leq l \leq 11\}$, respectively. The proposed design shows an unstable region at the design $\{(k, l) | 8.3 \leq k < 8.7, 6 \leq l \leq 11\}$, while the LSDM does not. A further analysis, as shown in Table 6.7, has confirmed that this unstable region does exist as the corresponding eigenvalues are positive. Thus, the proposed design method is demonstrated to be more effective than the existing method.

6.5.2.5 Robust Design The effectiveness of the proposed robust design method is verified when uncontrollable variations Δd randomly vary in $[-0.05\ d_0, 0.05\ d_0]$. The parameter variation domain is divided into nine subdomains for the multiple-model approach. The robust solution is $k = 15$ and $l = 11$. A total of 1000 samples are taken to compare variation of the transient response with respect to Δd.

The transient responses under the constant initial condition and Δd are shown in Figure 6.14. From Figure 6.14, it is clear that the transient responses of the state x_1 and x_2 are stable and robust under parameter variation.

Moreover, let the initial value randomly vary in [1, 1.2]. From Figure 6.15, it is clear that the transient responses of state x_1 and x_2 are also stable and robust under parameter variation.

6.5.2.6 Design Comparison Effectiveness of the proposed robust design method is demonstrated in comparison with the LREDM when Δd randomly varies in $[-0.05\ d_0, 0.05\ d_0]$. The performance comparison is shown in Table 6.8, where the transient response of the existing design ($k = 8.7$, $l = 7.5$) clearly has a larger performance variation than the proposed method under both constant and variable initial values. Thus, the proposed method is less sensitive to uncontrollable variation than the existing method.

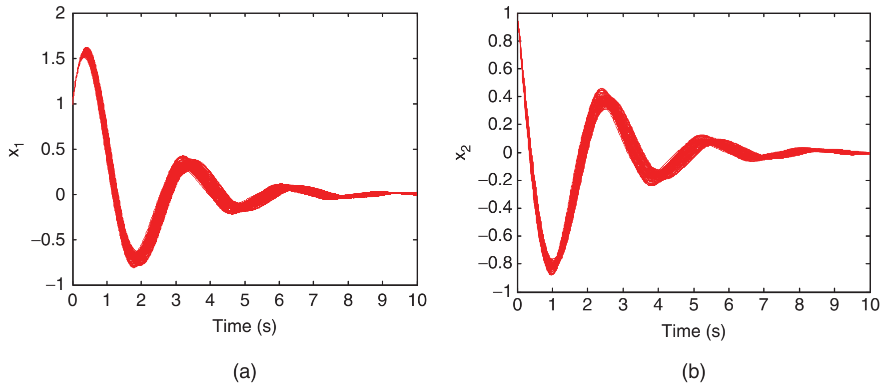


FIGURE 6.14 Performance under uncontrollable variations Δd: (a) transient response of $x_1(t)$; (b) transient response of $x_2(t)$

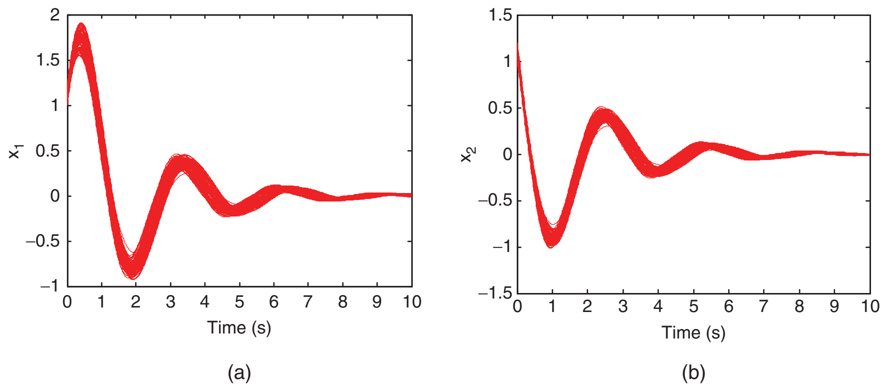


FIGURE 6.15 Performance under variation of initial condition and Δd: (a) transient response of $x_1(t)$; (b) transient response of $x_2(t)$

TABLE 6.8 Performance Comparison Under Different Design Methods

	Performance variation: $\Delta = \sqrt{\|x_1(t) - x_{10}(t)\|} + \|x_2(t) - x_{20}(t)\|$ (where x_{10}, x_{20} are response under nominal design parameter)			
	When the initial value is constant		When the initial value is variable	
	Mean	Variance	Mean	Variance
LSEDM	2.818	1.22	1.665	3.208
Proposed design method	1.511	0.209	0.189	1.302

6.6 SUMMARY

For small variation in the system, a linearization-based robust eigenvalue design method can effectively ensure the stability and robustness of a weak nonlinear dynamic system. First, a set of the design variables and bounds of their variations are figured out to guarantee system stability based on the stability theory and to be able to maintain the stability when design variables stay within these bounds. Then, in order to make the dynamic response less sensitive to variations, the robust eigenvalue design is considered to minimize the sensitivity of eigenvalues to parameter variations. Finally, tolerance space of robust design variables is maximized for the feasible design. An example is provided to demonstrate the effectiveness of the presented robust design method.

For large variations of uncontrollable parameters in the system, a multi-model-based robust design method is presented to ensure the stability as well as the robustness of the dynamic system. The multi-model approach could express the nonlinear relation well between dynamic performance and model parameters. The model built also has a linear structure that will be easy to design. Moreover, the developed design method has adequate ability to ensure the stability and dynamic robustness of the system even under large variations of uncontrollable parameters. The selected cases successfully demonstrate the effectiveness of the proposed method. The proposed method should be able to work under complex manufacturing environment.

CHAPTER 7

ROBUST EIGENVALUE DESIGN UNDER PARAMETER VARIATION—A NONLINEAR APPROACH

Chapter 6 takes into account the dynamic performance of linear system. This chapter will develop an approach to design stability and robustness of nonlinear system under large variations of uncontrollable parameters. A sector-nonlinearity (SN) method is first employed to model a nonlinear system. A stability design is then developed to ensure stability of this nonlinear system under parameter variations. The influence of parameter variations on eigenvalues of the system will also be minimized to maintain system robustness.

7.1 INTRODUCTION

The whole manufacturing process usually consists of many machines and systems. Due to physical and economic constraints, some functions or systems used in the process may not employ external controllers. Then, dynamic performance of these systems can only rely on system design. If the system can be designed to be inherently robust, a satisfactory performance will be easily achieved and insensitive to parameter variations. Clearly, robust design will be crucial to gaining a satisfactory dynamic performance.

Eigenvalues of the dynamic system play a crucial role in system design. The positions of eigenvalues critically dictate system stability, and their sensitivity to parameter variations will determine system robustness. Thus, both the positions and variations of eigenvalues are of vital importance for design of the dynamic system.

System Design and Control Integration for Advanced Manufacturing, First Edition. Han-Xiong Li and XinJiang Lu.

At the design stage, effort should be devoted to considering the effects of uncertainty on system stability. Significant research has been reported on this aspect (Blanco and Bandoni, 2003; Mohideen, Perkins, and Pistikopoulos, 1997; Kokossis and Floudas, 1994; Monnigmann and Marquardt, 2003, 2005). However, they may not work well for the system with strong nonlinearity, because the stability designed at the operation point may not work well in other location due to nonlinear characteristics (Liaw and Lee, 2006; Hachicho, 2007). Moreover, a good design should have minimum sensitivity of the system eigenvalues to parameter variations. Though a few studies have been presented recently to minimize the sensitivity of the eigenvalues with respect to parameter variations, such as methods in Chapter 6, linearization-based approach will generate a large approximation error for the strongly nonlinear system. In general, effective design is still needed to maintain both stability and dynamic robustness for the nonlinear dynamic system.

Generally, there are two significant factors that hinder the development of eigenvalue design in dynamic systems. First, system eigenvalues have a nonlinear relationship with model parameters, which is difficult to express explicitly (Blanco and Bandoni, 2003). Second, the eigenvalue design often has a nonlinear, even nonconvex or nondifferentiable objective function that is difficult to solve mathematically (Malcolm et al., 2007). Intelligent method, such as particle swam optimization (PSO) (Clerc 2006; Valle et al., 2008), would be a good choice to solve this complex optimization problem without analytical solutions.

In this chapter, a design method is proposed to achieve stability and robustness of a nonlinear dynamic system under parameter variations. A sector-nonlinearity (SN) method is first employed to model a nonlinear system. A stability design is then developed to ensure the nonlinear system to be stable in a desirable domain under variations. Furthermore, dynamic robustness will be achieved by minimizing the sensitivity of the system eigenvalues to parameter variations. This design optimization will be figured out by a newly constructed PSO-based two-loop optimization method. Finally, a practical example is used to demonstrate the effectiveness of the proposed method.

7.2 DESIGN PROBLEM

If external controllers are not used in manufacturing systems, the dynamic performance of such systems fully depends on their own design. Design of this kind of systems is not easy due to the strong nonlinearity and large parameter variation. The systems can be described as:

$$\dot{x} = f(x, d) \tag{7.1}$$

where $x \in R^n$ represents a state vector, $f(\cdot)$ is a nonlinear function, and $d = [d_1, \dots, d_m]^T$ is a design parameter vector whose nominal value is required to design. It is assumed that there is an uncontrollable variation Δd around the nominal value of this design parameter resulting from manufacturing and operating errors.

Stability Design

For a nonlinear dynamic system (Equation 7.1), the stability design needs to select the most appropriate design parameter to guarantee its asymptotic stability. The common design methods for the stability of the system (Equation 7.1), such as the methods in Chapter 6, are based on the following linearization model:

$$\dot{x} = A_0 x \tag{7.2}$$

with the constant nominal state matrix $A_0 = \left.\frac{\partial f}{\partial x}\right|_{x=x_0}$.

Then, a suitable design parameter is selected to have negative real parts of all eigenvalues of A_0. While these methods can obviously guarantee stability at the operating point, they cannot ensure stability in the whole state domain. A suitable stability domain will be crucial to robust performance of the nonlinear dynamic system (Hachicho, 2007; Liaw and Lee, 2006), as shown in Figure 7.1. Thus, further development is needed to achieve stability of a nonlinear dynamic system at a suitable domain instead of the operating point only.

Dynamic Robust Design

While parameter variations are unavoidable in practice, a dynamic robust design should be able to minimize its influence on a transient response. A small change of the transient response caused by parameter variations indicates a good robustness of the system. For the example shown in Figure 7.2, the design *b* is more robust than the design *a* because it has a smaller variation in the transient response.

According to the dynamic theory, the transient response of a system is decided by its eigenvalues (Liu and Patton, 1998; El-Kady and Al-Ohaly, 1997). Thus, the robust design of the dynamic system should have eigenvalues to be insensitive to large parameter variations. Although a few studies have been reported to minimize sensitivity of the eigenvalues to parameter variations based on the linearization model (Equation 7.2), such as the methods presented in Chapter 6, they are less effective to the nonlinear system since the neglected nonlinear terms still affect the system

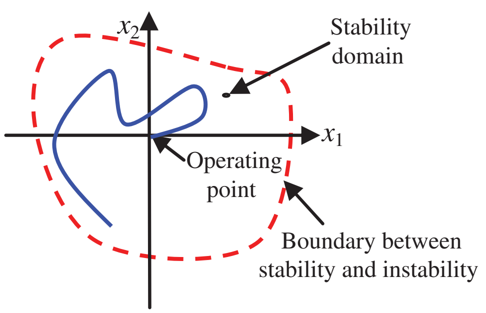


FIGURE 7.1 Stability domain of a nonlinear system

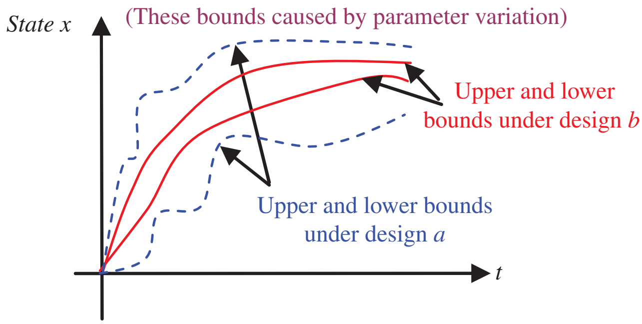


FIGURE 7.2 Variation bounds of transient response under different designs

response. Thus, effective design approach is required for the nonlinear system to have robust performance.

7.3 SN-BASED DYNAMIC DESIGN

Although sector-nonlinearity (SN) approach has often been used in the control field (Tanaka and Wang, 2001), it was hardly applied in design of a dynamic system except the case of static system design mentioned in Chapter 3. Here, it will be employed for the first time in designing a nonlinear dynamic system. The nonlinear system (Equation 7.1) will be transformed into the following form:

$$\dot{x} = f(x, d) = A(x, d)x \tag{7.3}$$

where the state matrix $A(x, d)$ is a function of x and d. This SN approach is illustrated in Figure 7.3 under the given design variable d. Any state derivative $\dot{x}$ may be expressed as the product of the state x and the state matrix $A(x, d)$. The state matrix $A(x, d)$ is equal to the slope of the straight line between its state point and the operating point O. For example, the state derivative $\dot{x}$ at point B is equal to $\dot{x}_B = A_{BO}x_B$, where the state matrix A_{BO} is equal to the slope of the line BO. Thus, any point of the

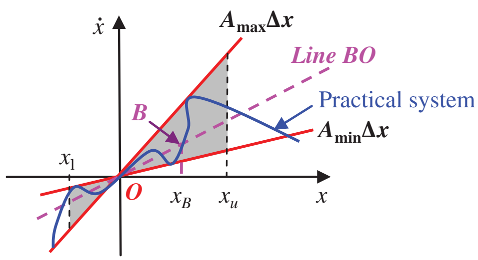


FIGURE 7.3 Sector nonlinearity modeling approach

nonlinear system may be expressed as $\dot{x} = A(x, d)x$. Since this model does not have any approximation, it can express the nonlinear system effectively.

Maximal and minimal values of the state matrix $A(x, d)$ within a given state domain $[x_l, x_u]$ can be easily calculated from the model, and expressed as:

$$A_{i,j}(x, d) \in [(A_{i,j}(d))_{\min}, (A_{i,j}(d))_{\max}] \tag{7.4}$$

with

$$\begin{aligned}(A_{i,j}(d))_{\min} &= \min_{x\in[x_l,x_u]} A_{i,j}(x, d)\\(A_{i,j}(d))_{\max} &= \max_{x\in[x_l,x_u]} A_{i,j}(x, d)\end{aligned}$$

where $A_{i,j}$ is the (i, j) element of A.

Example This simple example illustrates the mechanism of the SN method. The system is described as follows:

$$\begin{pmatrix}\dot{x}_1(t)\\\dot{x}_2(t)\end{pmatrix} = \begin{pmatrix}-x_1(t) + x_1(t)x_2^2(t)\\x_2(t) + x_1^3(t)x_2(t)\end{pmatrix} \tag{7.5}$$

with $x_1(t) \in [-1, 1]$ and $x_2(t) \in [-1, 1]$.

Equation 7.5 may be rewritten as:

$$\begin{pmatrix}\dot{x}_1(t)\\\dot{x}_2(t)\end{pmatrix} = \begin{bmatrix}-1 & x_1(t)x_2(t)\\1 & x_1^2(t)x_2(t)\end{bmatrix}\begin{pmatrix}x_1(t)\\x_2(t)\end{pmatrix} \tag{7.6}$$

Thus, its state matrix A is:

$$A = \begin{bmatrix}-1 & x_1(t)x_2(t)\\1 & x_1^2(t)x_2(t)\end{bmatrix} \tag{7.7}$$

Maximal and minimal values of its every element are easily estimated as:

$$(A_{1,2})_{\min} = \min_{x_1,x_2\in[-1,1]} x_1(t)x_2(t) = -1 \text{ and } (A_{1,2})_{\max} = \max_{x_1,x_2\in[-1,1]} x_1(t)x_2(t) = 1 \tag{7.8a}$$

$$(A_{2,2})_{\min} = \min_{x_1,x_2\in[-1,1]} x_1^2(t)x_2(t) = -1 \text{ and } (A_{2,2})_{\max} = \max_{x_1,x_2\in[-1,1]} x_1^2(t)x_2(t) = 1 \tag{7.8b}$$

7.3.1 Stability Design

Stability design is to ensure the nonlinear system to be stable in the given state domain by selecting a suitable design parameter. Here, a new approach for stability design is developed with two following issues to satisfy:

1. Stability design under the nominal parameter. This design finds a nominal stability parameter space S_n, which is composed of design parameters that guarantee the eigenvalues of the nominal matrix $A(x, d)$ to be negative.
2. Tolerance design. This design finds the tolerances of all design parameters in S_n so that the eigenvalues of $A(x, d + \Delta d)$ are negative only if their variations Δd are limited in the tolerances. From the viewpoint of robustness, these design parameters should have big tolerances.

Through these two designs, the system can be deemed stable when Δd is limited in its tolerance.

7.3.1.1 Stability Design Under the Nominal Parameter Stability of the nominal system in the given state domain $x \in [x_l, x_u]$ is designed without considering the influence of parameter variations. This design problem may be transformed to find design parameter d to make the Lyapunov function $\dot{V}(x)$ less than zero in accordance with the stability theory, where $V(x) = x^T P x$ and P is a symmetrically positive definite matrix:

$$
\begin{aligned}
\textit{Feasible}\ (P_1):\quad & \textit{Find}\ (d) \text{ to make} \\
& \dot{V}(x) = A^T(x, d)P + PA(x, d) < 0 \\
& P > 0, P = P^T \\
& A_{i,j}(x, d) \in [(A_{i,j}(d))_{\min}, (A_{i,j}(d))_{\max}] \\
& h(d) = 0,\ \ l(d) \le 0, x \in [x_l, x_u]
\end{aligned}
\tag{7.9}
$$

Where $h(d)$ and $l(d)$ are constraints from other design aspects.

The feasible problem P_1 is difficult to address directly, as it places an infinite number of inequalities corresponding to all the possible values of $A(x, d)$. According to Sandoval, Budman, and Douglas (2008), the feasible problem P_1 can be reduced to a finite set of Linear Matrix Inequalities (LMIs), where only a particular combination of the extreme values of all the elements at the state space matrix A is required:

$$
\begin{aligned}
\textit{Feasible}\ (P_2):\quad & \textit{Find}\ (d) \text{ to make} \\
& A_r P + P A_r < 0 \\
& (A_{i,j}(d))_{\min} \text{ or } (A_{i,j}(d))_{\max} \text{ is selected as the element } A_{r,(i,j)} \\
& P > 0, P = P^T \\
& h(d) = 0,\ \ l(d) \le 0
\end{aligned}
\tag{7.10}
$$

where the vector r denotes the set of 2^{mn} combinations. All possible solutions of the feasible problem (P_2) form a nominal stability parameter space S_n, in which any design parameter could ensure the stability of the nominal system in the given state domain $x \in [x_l, x_u]$.

7.3.1.2 Tolerance Design

This design finds tolerances of all of the design parameters in S_n. The nonlinear system (Equation 7.1) remains stable as $x \in [x_l, x_u]$ if every parameter variation Δd_i is limited in its tolerance D_i, even $D_i = \max(|\Delta d_i|)$ $(i = 1, \ldots, m)$. Such a tolerance may be obtained by solving the following feasible problem (P_3):

$$\begin{aligned}
\textit{Feasible } (P_3)\text{:}\quad & \textit{Find } (D_1, \ldots, D_m) \text{ to make} \\
& \tilde{A}_r\ P + P\tilde{A}_r\ < 0 (\tilde{A}_{i,j}(d))_{\min} \text{ or } (\tilde{A}_{i,j}(d))_{\max} \\
& \quad \text{is selected as the element } \tilde{A}_{r,(i,j)} \\
& (\tilde{A}_{i,j}(d))_{\min} = \min_{\substack{x\in[x_l,x_u] \\ \Delta d\in[-D,D]}} (A_{i,j}(x, d + \Delta d)) \\
& (\tilde{A}_{i,j}(d))_{\max} = \max_{\substack{x\in[x_l,x_u] \\ \Delta d\in[-D,D]}} (A_{i,j}(x, d + \Delta d)) \\
& D_i = \max(|\Delta d_i|), \\
& P > 0, P = P^T \\
& h(d + \Delta d) = 0, l(d + \Delta d) \le 0 \\
& d \in S_n
\end{aligned} \tag{7.11}$$

From a robustness view, the larger the tolerance is, the better the stability is. In this sense, the tolerance D_i should be larger than the desirable level ε_i. To be precise, such a design parameter d in S_n should be accepted only if its tolerance is larger than the desirable level. All accepted design parameters form a stability parameter space S_s.

7.3.2 Dynamic Robust Design

The relationship between the eigenvalue and its associated eigenvector can be expressed as follows

$$A(x, d + \Delta d)U = U\Lambda \tag{7.12}$$

where $U = [u_1, \ldots, u_n]$ and $\Lambda = diag[\lambda_1, \ldots, \lambda_n]$ are eigenvector matrix and eigenvalue matrix, respectively. While the state matrix A in the SN method is variable, its eigenvalue λ_i is also variable in a bound:

$$\lambda_i \in \left[\lambda_i^{\min}, \lambda_i^{\max}\right] \tag{7.13}$$

It is well known that the transient response of the system can be expressed as:

$$x(t) = Ue^{\Lambda t}U^{-1}x(0) \tag{7.14}$$

where $x(0)$ is an initial value. From Equation 7.14, the time response $x(t)$ is mainly decided by the eigenvalue λ_i of the state matrix A. If the eigenvalue λ_i is sensitive to parameter variations, the dynamic behavior will deviate from the desired response. This is undesirable for a practical engineering system. Thus, a dynamic robust design should be developed by choosing a suitable design variable d from the stability parameter space S_s to make all eigenvalues less sensitive to parameter variations.

According to the orthogonal theory of eigenvector, the system eigenvalue λ_i ($i = 1, \ldots, n$) is shown below with the detailed derivation in Chapter 6:

$$\lambda_i(x, d + \Delta d) = \lambda_i(x, d) + \left(v_i^T(x, d) \frac{\partial A(x, d + \Delta d)}{\partial \Delta d} \bigg|_{\Delta d = 0} u_i(x, d) \right) \Delta d \tag{7.15}$$

where v and u are the left and the right eigenvector of the eigenvalue λ_i, respectively. Define

$$\Delta\lambda_i = \lambda_i(x, d + \Delta d) - \lambda_i(x_0, d) \tag{7.16}$$

From Equations 7.15 and 7.16, we have

$$\Delta\lambda_i = \lambda_i(x, d) - \lambda_i(x_0, d) + \left(v_i^T(x, d) \frac{\partial A(x, d + \Delta d)}{\partial \Delta d} \bigg|_{\Delta d = 0} u_i(x, d) \right) \Delta d \tag{7.17}$$

From Equation 7.17, the eigenvalue variation $\Delta\lambda_i$ is simultaneously affected by both the parameter variation Δd and the nonlinear term of the system (Equation 7.1). This creates a challenge in minimizing $\Delta\lambda_i$ as a result of complexity and interaction.

Here, a design approach (Figure 7.4) is proposed to decompose this complex design problem (Equation 7.17) into two simple subproblems. One is to minimize

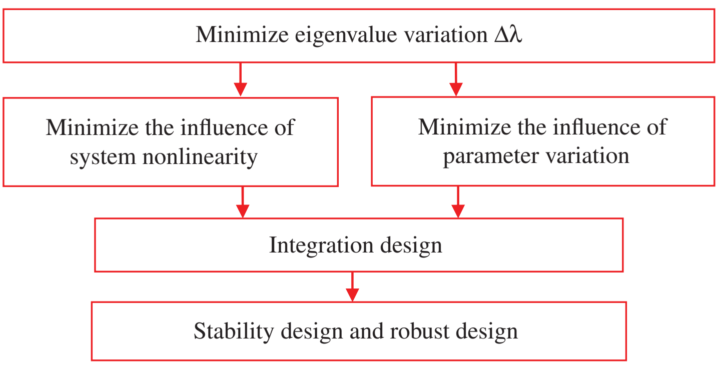


FIGURE 7.4 New robust design method

the influence of the system nonlinearity to eigenvalue variation ($\Delta\lambda/x$). The other is to minimize the influence of parameter variations on eigenvalue variation ($\Delta\lambda/\Delta d$). Finally, these two subproblems are integrated into a unified optimization framework, where robust solution can be achieved in the given state domain.

7.3.2.1 Minimizing the Influence of System Nonlinearity First of all, the first and second term $\lambda_i(x, d) - \lambda_i(x_0, d)$ on the right side of Equation 7.17 are minimized. Even though these terms are related to the nonlinearity of the system, they are not related to Δd. According to the developed SN model, the maximal variation of eigenvalues can be obtained in the given state domain through the following optimization.

$$\Delta\lambda_{i,x}^{\max} = \max_{x_l<x<x_u} |\lambda_i(x, d) - \lambda_i(x_0, d)| \tag{7.18}$$

In order to minimize the nonlinear influence on the system eigenvalues, a suitable design variable d will be chosen to minimize this maximal eigenvalue $\Delta\lambda_{i,x}^{\max}$:

$$\begin{aligned} & C_1(d) \\ & \min_d \sum_{i=1}^{m} \Delta\lambda_{i,x}^{\max} \\ & \text{s.t. } d \in S_s \end{aligned} \tag{7.19}$$

where $d \in S_s$ is the requirement for stability and feasibility.

7.3.2.2 Minimizing the Influence of Δd In addition, the other term $(v_i^T(x, d)\frac{\partial A(x,d+\Delta d)}{\partial \Delta d}\big|_{\Delta d=0}\, u_i(x, d))\Delta d$ on the right side of Equation 7.17 is minimized. This can be defined as

$$\Delta\lambda_{i,d} = \left(v_i^T(x, d)\frac{\partial A(x, d + \Delta d)}{\partial \Delta d}\bigg|_{\Delta d=0} u_i(x, d)\right)\Delta d \tag{7.20}$$

For convenience, $v(x, d)$ and $u(x, d)$ can be denoted simply as v and u. Obviously, Equation 7.20 may be rewritten as:

$$\Delta\lambda_d = J\psi \tag{7.21}$$

where

$$\Delta\lambda_d = \begin{bmatrix} \Delta\lambda_{1,d} \\ \vdots \\ \Delta\lambda_{m,d} \end{bmatrix}, \quad \psi = \begin{bmatrix} \Delta d_1 \\ \vdots \\ \Delta d_m \end{bmatrix},$$

$$J = \begin{bmatrix} \left(v_1^T \dfrac{\partial A(x, d + \Delta d)}{\partial \Delta d_1}\mu_1\right)\bigg|_{\Delta d=0} & \cdots & \left(v_1^T \dfrac{\partial A(x, d + \Delta d)}{\partial \Delta d_m}\mu_1\right)\bigg|_{\Delta d=0} \\ \vdots & \ddots & \vdots \\ \left(v_n^T \dfrac{\partial A(x, d + \Delta d)}{\partial \Delta d_1}\mu_n\right)\bigg|_{\Delta d=0} & \cdots & \left(v_n^T \dfrac{\partial A(x, d + \Delta d)}{\partial \Delta d_m}\mu_n\right)\bigg|_{\Delta d=0} \end{bmatrix}.$$

From Equation 7.21, it is clear that ψ represents variation of the design parameter, while J is only affected by x. Since the designer cannot control the term ψ, this robust design problem is reduced to the minimization of its sensitivity matrix J. Thus, performance will directly relate to the design parameter as follows:

$$\|\mathrm{Re}(\Delta\lambda_d)\|_2^2 + \|\mathrm{Im}(\Delta\lambda_d)\|_2^2 = \sum_{i=1}^{m} \sigma_i y_i^2 \tag{7.22}$$

with $[y_1, \dots, y_m]^T = V^T \psi$.
where σ_i is the singular value of $\mathrm{Re}(J)^T\mathrm{Re}(J) + \mathrm{Im}(J)^T\mathrm{Im}(J)$, and the corresponding orthogonal eigenvector is denoted as V_i, which is one element of $V = [\, V_1 \; \cdots \; V_m \,]$.

The performance $\Delta\lambda_d$ will be less sensitive to parameter variation Δd if the maximal singular value $\sigma_{\max}$ is minimized, according to sensitivity-based robust design approaches. Thus, in order to minimize $\Delta\lambda_d$, the design variable d should be chosen to minimize the maximal singular value $\sigma_{\max}$ as: $\Delta\lambda_{i,x}^{\max}$:

$$C_2(d)$$

$$\begin{aligned} &\min_{d} \max_{x_l<x<x_u} (\sigma_i) \\ &s.t. \; \|\mathrm{Re}(\Delta\lambda_d)\|_2^2 + \|\mathrm{Im}(\Delta\lambda_d)\|_2^2 = \sum_{i=1}^{m} \sigma_i y_i^2 \\ &d \in S_s \end{aligned} \tag{7.23}$$

Therefore, the solution of Equation 7.23 can minimize the influence of parameter variation on $\Delta\lambda_d$.

7.3.2.3 Integration Design This design simultaneously considers influences from both the nonlinear term and parameter variations, where two objective functions C_1 in Equation 7.19 and C_2 in Equation 7.23 are integrated into a unified objective function using the weighted-sum method:

$$C_3(d): \quad \begin{aligned} &\min_{d} \; \beta \frac{C_1(d)}{C_1(d^+)} + (1-\beta)\frac{C_2(d)}{C_2(d^+)} \\ &s.t. \; d \in S_s \end{aligned} \tag{7.24}$$

where the objectives $C_1(d)$ and $C_2(d)$ are normalized by their central values $C_1(d^+)$ and $C_2(d^+)$ with the central point d^+ of d, while β is a trade-off weight in the range $0 \prec \beta \prec 1$.

A desirable robust design can be obtained by solving the integration problem (Equation 7.24) with an optimal weight factor β^*. However, as the singular σ has a complex nonlinear relation with the design parameter, the integration problem (Equation 7.24) is often a nonconvex and nonlinear constrain optimization problem.

Moreover, this nonlinear relation is often difficult to express mathematically. As a result, the integration problem can be difficult to solve by many analytical methods.

7.3.2.4 A PSO-Based Two-Loop Optimization Method Here, a PSO-based two-loop optimization method is applied to address this problem. This method decomposes the integration problem into two nested optimizations: optimization of design parameter (inner loop) and optimization of weight factor (outer loop).

1. Optimization of design parameter (inner loop)

This optimization solves the integration problem (Equation 7.24) when the weight factor β is given. This optimization strategy is shown in Figure 7.5 and summarized as follows:

Step 1: The weight factor β is offered by optimization of weight factor (outer loop) and the design variable d is initialized.

Step 2: $C_1(d)$, $C_2(d)$, and the integration design cost (Equation 7.24) are computed.

Step 3: Judge whether the integration design cost (Equation 7.24) satisfies the requirement. If it is satisfactory, then the design will be accepted and the optimization design is finished. Otherwise, the design variable d must be updated. Here, the PSO method as presented in next section is employed to update the design variable d, and then the program goes to Step 2.

Step 4: Steps 2–3 are repeated until a stopping criterion is met.

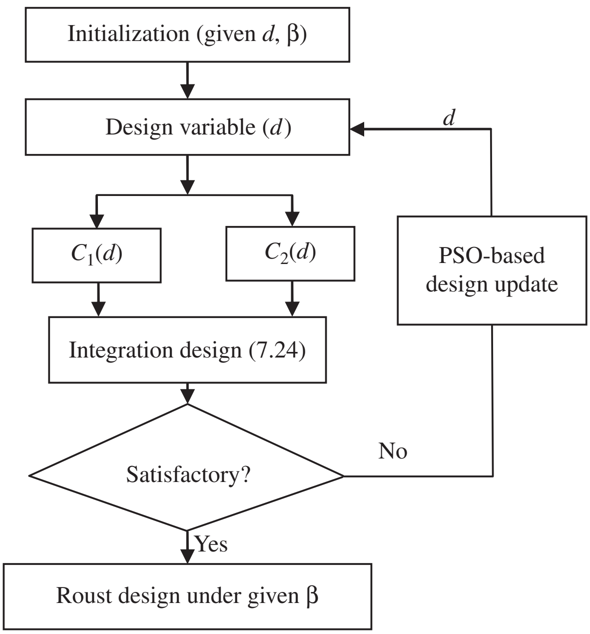


FIGURE 7.5 Optimization of design parameter (inner loop)

In the particle swarm optimization, each individual possible solution can be modeled as a particle that moves through the problem hyperspace (Eberhart and Kennedy, 1995). The position of the ith particle at a certain iteration time t is determined by the vector of the coordinates

$$d_i(t) = [d_{1i}(t), \dots, d_{\beta i}(t)] \tag{7.25}$$

where the component $d_{ji} \in [d_{ji}^{\min}, d_{ji}^{\max}]$ represents the jth design variable of the ith particle, and β is the number of design variables namely the dimension of the search space.

The information available for each individual is based on its own experience and the knowledge of other individuals. Since the relative importance of these two factors can vary from one decision to another, it is reasonable to apply random weights to each part. Therefore, the velocity v_i of the ith particle will be determined by

$$\begin{aligned} v_i(t) = \delta v_i(t-1) &+ \varphi_1 \cdot rand_1 \cdot (p_{best,i}(t-1) - d_i(t-1)) \\ &+ \varphi_2 \cdot rand_2 \cdot (g_{best,i}(t-1) - d_i(t-1)) \end{aligned} \tag{7.26}$$

where δ is an inertia weight, φ_1 and φ_2 are two positive numbers, and $rand_1$, $rand_2$ are two random numbers with uniform distribution in range of [0, 1], and $p_{best,i}$ and $g_{best,i}$ give the best position of the ith particle and the entire swarm, respectively.

Then, the position of the ith particle is updated according to

$$d_i(t+1) = d_i(t) + v_i(t) \tag{7.27}$$

When the value of a coordinate of a particle lies outside the acceptable interval, it means that the particle leaves the search space. The velocity and position of this particle should be modified to bring it back inside the search space (Naka et al., 2003)

$$d_{ji} \notin [d_{ji}^{\min}, d_{ji}^{\max}] \Rightarrow \begin{cases} v_{ji}(t) = 0 \\ d_{ji} \prec d_{ji}^{\min} \Rightarrow d_{ji} = d_{ji}^{\min} \\ d_{ji} \succ d_{ji}^{\max} \Rightarrow d_{ji} = d_{ji}^{\max} \end{cases} \tag{7.28}$$

where $v_{ji}(t)$ is the jth element of $v_i(t)$.

The following procedure can be used for implementing the PSO algorithm (Clerc, 2006):

Step 1: Initialize the swarm by assigning a random position in the problem hyperspace to each particle.

Step 2: Evaluate the fitness function $C_3(d)$ for the ith particle and judge the feasibility of the ith particle. The feasibility test is to check all constraint conditions. If all constraint conditions are feasible, then such a particle will be accepted. Otherwise, this particle is unaccepted and then the program goes to Step 5.

Step 3: For each individual particle, compare the particle's fitness value $C_3(d)$ with its $p_{best,i}$. If the current value is better than $p_{best,i}$ value, then set this value as $p_{best,i}$ and the current position of the particle d_i as p_i.

Step 4: Identify the particle with the best fitness value. The value of its fitness function is identified as $g_{best,i}$ and its position as p_g.

Step 5: Update the velocities and position of all the particles using Equations 7.26 and 7.27. Then, judge whether the particle leaves the search space. If yes, then carry out Equation 7.28.

Step 6: Repeat Steps 2–5 until a stopping criterion is met. Finally, the optimal design variables are taken from p_g.

2. Optimization of weight factor (outer loop)

This optimization is developed as shown in Figure 7.6 to find the optimal weight factor. It mainly includes the following steps:

Step 1: The weight factor β is initialized.

Step 2: The optimization of design parameter (inner loop) is performed.

Step 3: Check whether its solution satisfies the robust performance. If it is satisfactory, then the design is considered robust and the optimization design is finished. Otherwise, the weight factor β needs to be updated, and then the program goes to Step 2. Here, the PSO method is also used for the updating.

Step 4: Steps 2–3 are repeated until a stopping criterion is met.

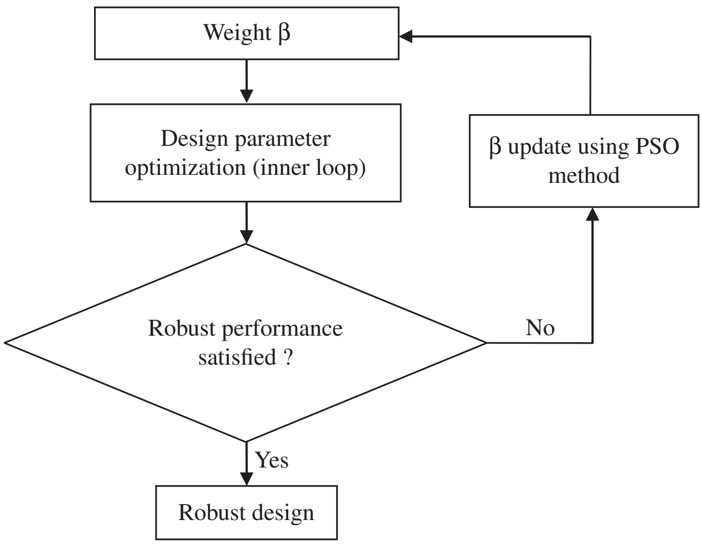


FIGURE 7.6 Optimization of weight factor (outer loop)

7.3.2.5 Design Summary Many manufacturing systems are often strongly nonlinear and their model parameters have a large variation domain. The proposed design will be able to maintain the system stability as well as the system robustness. Its design procedure is summarized as follows:

Step 1: A nonlinear system is modeled using the SN approach. Since this modeling has an equivalent expression as the original system, it will have less approximation than the traditional linearization approach in a large state region.

Step 2: The nominal stability parameter space S_n is found by solving the feasible problem (P_2). The stability of the nominal nonlinear system can thus be guaranteed.

Step 3: The stability parameter space S_s is found within the nominal stability parameter space S_n by solving the feasible problem (P_3). When variation of design parameter from S_s is limited in its tolerance, its stability is still maintained.

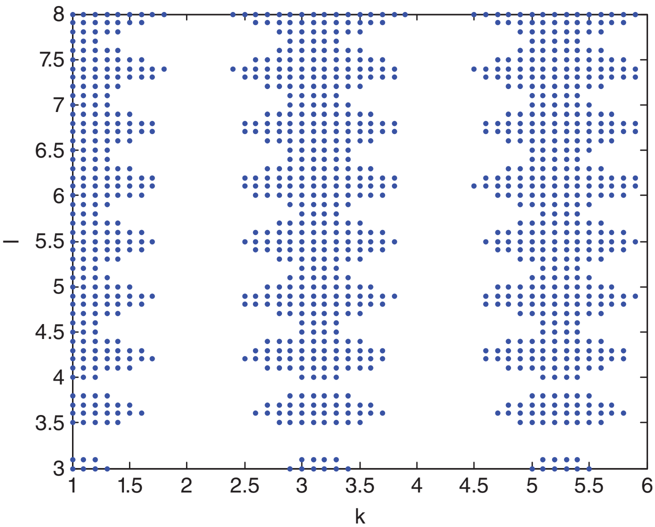
Step 4: The dynamic robust design from S_s is obtained by solving the optimization problem (Equation 7.24) using the PSO-based two-loop optimization method. Since it considers the influences from both the nonlinear term and parameter variations, it will be more effective in obtaining the desirable dynamic response of a nonlinear system under large parameter variations.

7.4 CASE STUDY

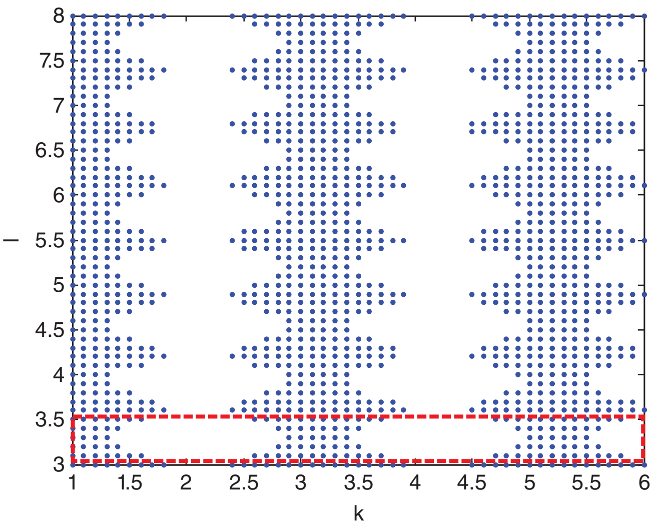
A magnetic actuator example with a variable working point as presented in Chapter 6 is used to verify the effectiveness of the proposed method. In this magnetic actuator design, the design parameters are k and l, with nominal values chosen from their design space $k \in [1, 6]$ and $l \in [3, 8]$. However, there are uncontrollable variations Δk and Δl around their nominal values. These variations could come from magnetic leak and manufacturing error. A key issue for high precision actuator is to choose suitable design parameters to ensure both stability and robustness of the system within the given state domain $x_1(t) \in [-2.5, 2.5]$ and $x_2(t) \in [-2.5, 2.5]$ under uncertainty.

7.4.1 Stability Design

Stability domains obtained by the proposed stability design method and the linearization-based robust eigenvalue design as presented in Chapter 6 are compared under the desirable level $\varepsilon_k = \varepsilon_l = 0.01$. As shown in Figure 7.7, the white blank space represents the unstable designs while the blue shadow space represents the stable designs. The proposed design can identify an unstable region at bottom around the parameter $l = 3.25$, while the linearization-based robust eigenvalue design fails. A further analysis, as shown in Table 7.1, has confirmed that this unstable region is correct as the corresponding eighenvalues (λ_1, λ_2) are positive. Thus, the proposed design method is demonstrated to be more effective than the linearization-based robust eigenvalue design method because of less approximation in modeling.



(a)



(b)

FIGURE 7.7 Stability design under different design methods: (a) stability design obtained by the proposed method; (b) stability design obtained by linearization-based robust eigenvalue design method

TABLE 7.1 Unstable Solution at Design Parameter $l = 3.25$

k	L	x_1	x_2	λ_1	λ_2
1	3.25	2.5	−2.5	0.75	0.75
2	3.25	0	0	0.67	7.41
3	3.25	6	3	0.64	0.64
4	3.25	−0.5	0.5	0.94	7.43
5	3.25	−2.5	2.5	0.38	0.38
6	3.25	0	0	2.09	4.29

7.4.2 Dynamic Robust Design

Ten particles are employed for the PSO method, respectively, in the inner loop and the outer loop. The iterative process of the outer loop is shown in Figure 7.8. The developed PSO-based two-loop optimization method is used to obtain the robust design solution d_I^* ($k^* = 3.153$, $l^* = 8$, $\beta^* = 0.795$) for the design problem (Equation 7.24).

The performance comparison with the existing method, the linearization-based robust eigenvalue design in Chapter 6, demonstrates the effectiveness of the proposed robust design method when both Δk and Δl vary randomly in [−0.01, 0.01]. A total of 1000 samples from random variation of Δk and Δl are used and every sample produces a corresponding transient response.

Upper and lower bounds among 1000 transient responses are shown in Figure 7.9 under the consistent initial condition. From Figure 7.9, it is evident that the transient responses of state x_1 and x_2 obtained by the existing design have a larger variation than the proposed method.

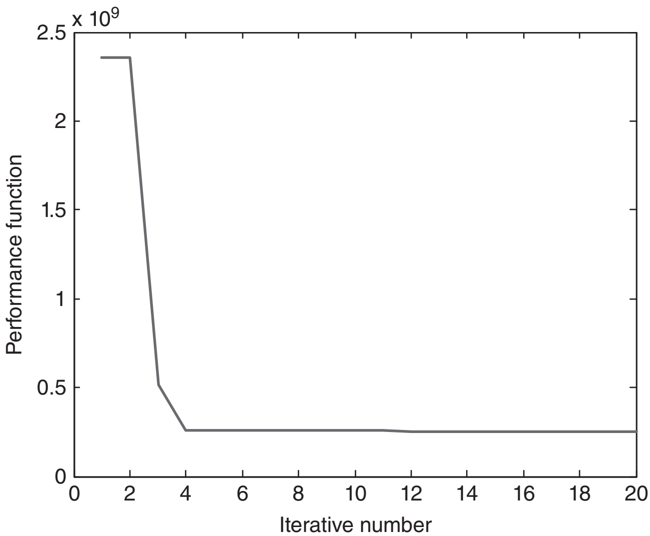


FIGURE 7.8 Iterative process in the outer loop

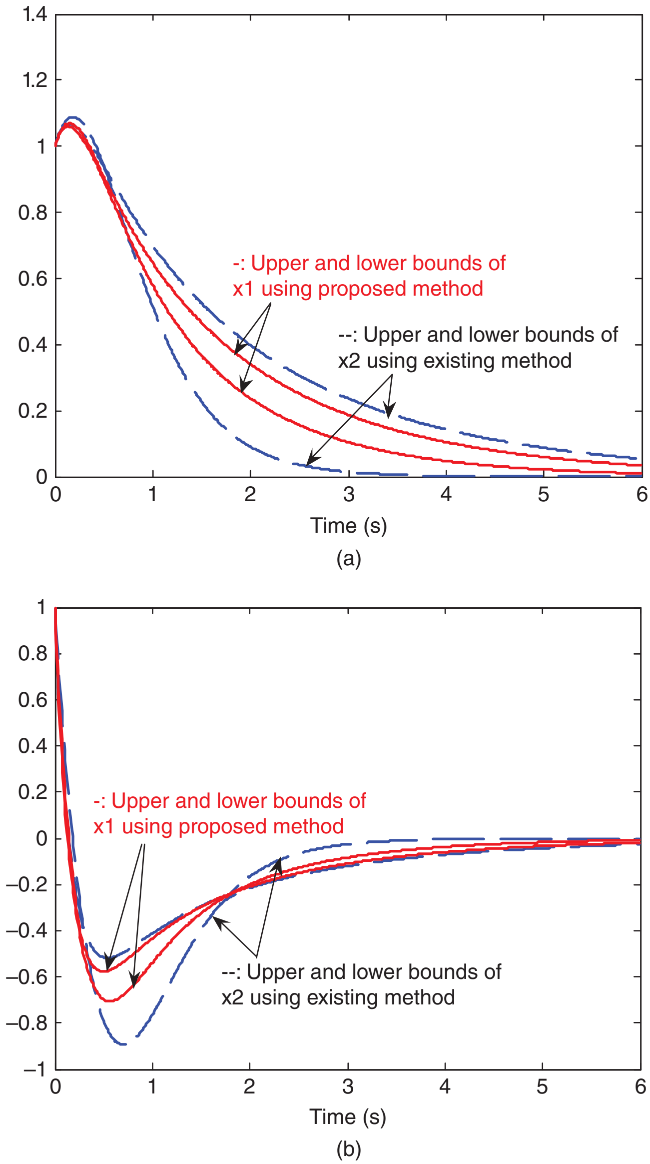


FIGURE 7.9 Performance comparison under Δk and Δl: (a) transient response of $x_1(t)$; (b) transient response of $x_2(t)$

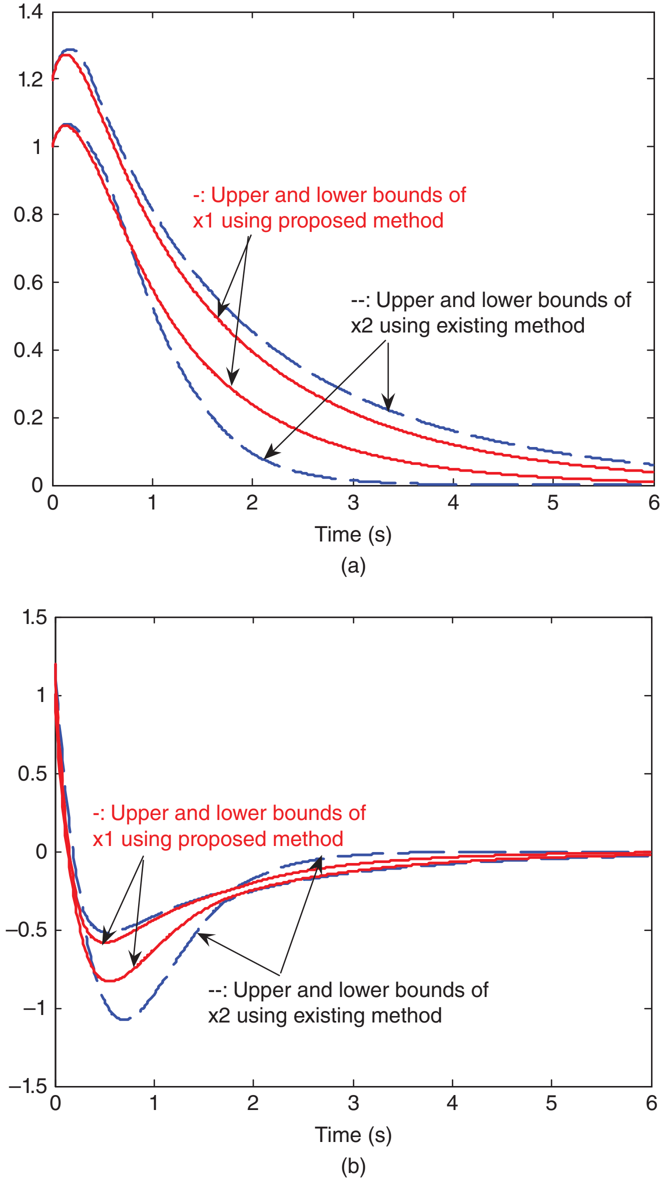


FIGURE 7.10 Performance comparison under variation of initial condition and Δk and Δl; (a) transient response of $x_1(t)$; (b) transient response of $x_2(t)$

The initial value is also allowed to vary randomly in [1, 1.2]. Under the variation of the initial value, the upper and lower bounds obtained by the proposed method and the existing method are compared in Figure 7.10. The transient response of the existing design clearly has a larger performance variation than the proposed method.

Thus, the proposed method is less sensitive to parameter variations than the existing method, because it effectively considers the influence of the nonlinear term in large parameter variation, while the existing method does not.

7.5 SUMMARY

A design method is proposed for the nonlinear dynamic system for improving stability and robustness. The SN method provides a well-designed model for the nonlinear system working in a large state region. Thus, this novel design method could achieve both stability and dynamic robustness of the system under large parameter variations. The selected case on design of a practical nonlinear system demonstrates the effectiveness of the proposed method. It would be a more realistic method for design of the manufacturing system.

CHAPTER 8

ROBUST EIGENVALUE DESIGN UNDER MODEL UNCERTAINTY

The methods presented in Chapter 6 and Chapter 7 are to minimize the effect of parameter variations on dynamic performance without considering the influence of model uncertainty. This chapter will develop a novel approach to design the system to be stable and robust under parameter variations as well as model uncertainty. With the help of the known model information and data-based uncertainty estimation, design variables and their variation bounds are first configured to make the system stable. A perturbation theory-based robust design is then developed to make the dynamic response less sensitive to parameter variations as well as model uncertainty.

8.1 INTRODUCTION

Due to complex dynamics and boundary conditions in manufacturing, it is often difficult to obtain accurate models for many systems used in manufacturing production. A nominal model, derived from simplicity and ideal assumptions, is often used to approximate the original system. For example, fluid process, like epoxy dispensing (Hong and Li, 2003) and snap curing (Deng, Li, and Chen, 2005) in IC/LED packaging industry, are generally described in a nonlinear partial differential equation with complex boundary conditions, which are difficult to solve analytically. Thus, they are often simplified into a set of nominal differential equations under some assumptions (Li and Qi, 2010, 2011). This simplification will cause model uncertainty, in either

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parameter or structure. If these uncertainties are not considered in design, system performance will be deteriorated. Thus, designers should seriously consider the effect of model uncertainty on system performance.

In recent several decades, much effort has been devoted to consider system dynamics at the process design stage, for example, chemical process design (Blanco and Bandoni, 2003) and rotor systems design (Kliem, Pommer, and Stoustrup, 1998). However, these studies still have their limitations: (a) they paid no attention to the effect of model uncertainty on system stability at the stage of process design; and (b) they did not consider the influence of model uncertainty on dynamic response and thus the designed system was often less robust.

In this chapter, a design method is developed to achieve the desired dynamic performance under model uncertainty. First, the stability theory is applied to obtain a feasible space for design variables under model uncertainty. When design variables stay within this feasible space, the system is still stable even if both model uncertainty and fluctuations of design variables exist. Then, the robust eigenvalue design is developed to minimize the effect of uncertainties on system eigenvalues using the matrix perturbation theory, which will guarantee the dynamic response less sensitive to uncertainties. Furthermore, the tolerance space of the obtained robust design will be maximized under specified performance constraints. Finally, an example is provided to demonstrate the effectiveness of the proposed approach.

8.2 DESIGN PROBLEM FOR PARTIALLY UNKNOWN DYNAMIC SYSTEMS

Consider the model uncertainty design problem:

$$\dot{x} = f(x, d) + \Delta f(x, d) \tag{8.1}$$

where $x \in R^n$ represents state vector, $d = [d_1 \cdots d_m]^T$ is design variable vector, $f(d) = [f_1(d) \cdots f_n(d)]^T$ is known nominal model, $\Delta f(d) = [\Delta f_1(d) \cdots \Delta f_n(d)]^T$ is model uncertainty.

Taking Taylor series expansion of the model (Equation 8.1) at $x = 0$ and neglecting the high-order term, the system model can be expressed as

$$\dot{x} = A(d)x \tag{8.2}$$

where the nominal Jacobian matrix $A_0(d)$, the perturbation Jacobian matrix $\Delta A(d)$, and the Jacobian matrix $A(d)$ are defined as

$$A_0(d) = \frac{\partial f(x, d)}{\partial x}\Big|_{x=0} \tag{8.3a}$$

$$\Delta A(d) = \frac{\partial \Delta f(x, d)}{\partial x}\Big|_{x=0} \tag{8.3b}$$

$$A(d) = A_0(d) + \Delta A(d) \tag{8.3c}$$

Since Δf is unknown, ΔA is also unknown. For convenience, $A_0(d)$, $\Delta A(d)$, and $A(d)$ are simply denoted as A_0, ΔA, and A respectively.

Obviously, stability is critical to the dynamic system. The objective of the stability design is to select the design variables d to make the real parts of λ smaller than zero. However, since there is model uncertainty in the system, it is difficult to directly obtain λ, which poses a challenge to design.

Moreover, since variations of these design variables are unavoidable, their variation bounds, around their nominal values, should be calculated to restrain their influences on stability. Due to simultaneous existence of model uncertainty and variations of design variables, it is difficult to obtain these bounds, which will pose another challenge to design.

Furthermore, under the guaranteed system stability, how to make the system dynamic response less sensitive to both parameter variations and model uncertainty is the third challenge to deal with.

8.3 STABILITY DESIGN

In this section, a stability design method is developed to guarantee the system stability under parameter variations as well as model uncertainty.

8.3.1 Stability Design for Nominal Model

The objective of this design is to choose such design variables d to make the real part of all eigenvalues $\lambda^0 = [\lambda_1^0\ \lambda_2^0 \cdots \lambda_n^0]$ of the nominal Jacobian matrix A_0 smaller than zero. These design variables d can be figured out by the following feasible problem (P_1):

$$\begin{aligned} \textit{Feasible } (P_1)\text{:}\quad & \textit{Find } (d) \\ & \textit{to make} \\ & \mathrm{Re}(\lambda_{\mathrm{i}}(A_0(d))) < 0\ (i = 1, \ldots,\ n) \\ & h(d) = 0, l(d) \le 0 \end{aligned} \tag{8.4}$$

where $h(d)$ and $l(d)$ are constraints from other design aspects, and $\mathrm{Re}(\lambda)$ denotes the real part of λ. All possible solutions of the feasible problem (P_1) form a nominal stability variable space S_n, in which every design variable will guarantee the stability of the nominal system.

8.3.2 Stability Design Under Model Uncertainty

The eigenvalues $\lambda_i^0 (i = 1, \ldots, n)$ of A_0 are related to its eigenvectors $U_i^0\ (i = 1, \ldots, n)$ by the equations

$$A_0 U_0 = U_0 \Lambda_0 \tag{8.5}$$

where $U_0 = [U_1^0, \ldots, U_n^0]$ and $\Lambda_0 = diag(\lambda_1^0, \ldots, \lambda_n^0)$ are the right eigenvector set and the right eigenvalue set of A_0, respectively.

According to the Bauer–Fike theorem (Stewart and Sun, 1990; Ralph and Stephen, 1989), if A_0 has an additive perturbation ΔA, then a bound on the variations of the eigenvalues is given by

$$\left|\lambda_i - \lambda_i^0\right| \le K \cdot \|\Delta A\|_2 \tag{8.6}$$

where λ_i and λ_i^0 are the eigenvalues of A and A_0 respectively, the condition number K is the ratio of the largest singular value $\sigma_{U\max}$ to the smallest singular value $\sigma_{U\min}$ of U_0.

The upper bound S of the model uncertainty is estimated as

$$\|\Delta A\|_2 \le S \tag{8.7}$$

This is easy to ascertain by means of system knowledge, experience, or sample data, such as, the method in Chapter 5 for estimation of this upper bound from data. This estimation can effectively compensate the effect of model uncertainty on system performance.

Then, from inequalities (Equations 8.6 and 8.7), we obtain

$$\left|\lambda_i - \lambda_i^0\right| \le K \cdot S \tag{8.8}$$

Inequality (Equation 8.8) may be rewritten as

$$\mathrm{Re}(\lambda_i^0) - K \cdot S \le \mathrm{Re}(\lambda_i) \le \mathrm{Re}(\lambda_i^0) + K \cdot S \tag{8.9}$$

Theorem 8.1 Consider system (Equation 8.1) with model uncertainty. Given the nominal stability variable space S_n and the condition (Equation 8.7), if $K \cdot S$ is smaller than $|\mathrm{Re}(\lambda_i^0)|$, then all eigenvalues of A have negative real parts (A is a Hurwitz matrix) and system (Equation 8.1) is asymptotically stable even if model uncertainty exists.

Proof: Since all design variables are within the nominal stability variable space S_n, $\mathrm{Re}(\lambda_i^0)$ is smaller than zero. From the inequality (Equation 8.9), if $K \cdot S$ is smaller than$|\mathrm{Re}(\lambda_i^0)|$, the upper bound $\mathrm{Re}(\lambda_i^0) + K \cdot S$ of $\mathrm{Re}(\lambda_i)$ will be smaller than zero. Therefore, system (Equation 8.1) is asymptotically stable.

Thus, the stability design for model uncertainty can be obtained by solving the following feasible problem (P_2):

$$\begin{aligned} \textit{Feasible } (P_2)\text{:} \quad & \textit{Find } (d) \\ & \textit{to make} \\ & K \cdot S < \left|\mathrm{Re}\left(\lambda_i^0\right)\right| (i = 1, \ldots, n) \\ & d \in S_n \end{aligned} \tag{8.10}$$

According to the above proof, the inequalities $K \cdot S < |\mathrm{Re}(\lambda_i^0)|$ $(i = 1, \ldots, n)$ will guarantee all eigenvalues to be negative even if model uncertainty exists in the system. All possible solutions of the feasible problem (P_2) form a stability variable space S_s, in which any design variable will guarantee system stability even if model uncertainty exists.

8.3.3 Stability Bound of Design Variables

When the design variables d have small variations Δd, the Jacobian matrix A becomes

$$A(d + \Delta d) = A_0(d + \Delta d) + \Delta A(d + \Delta d) \tag{8.11}$$

Taking Taylor series expansion of $A_0(d + \Delta d)$ and neglecting the high-order term, the Jacobian matrix $A(d + \Delta d)$ may be rewritten as

$$A(d + \Delta d) = A_0(d) + \Delta \tilde{A} \tag{8.12}$$

with $\Delta \tilde{A} = \frac{\partial A_0(d+\Delta d)}{\partial \Delta d}\big|_{\Delta d=0} \Delta d + \Delta A(d + \Delta d)$.

According to the Bauer–Fike theorem and the matrix norm theory, we have

$$\begin{aligned} |\tilde{\lambda}_i - \lambda_i^0| &\le K \|\Delta \tilde{A}\|_2 \\ &= K \cdot \left\| \frac{\partial A_0(d + \Delta d)}{\partial \Delta d} \Big|_{\Delta d=0} \Delta d + \Delta A(d + \Delta d) \right\|_2 \\ &\le K \cdot \left(\left\| \frac{\partial A_0(d + \Delta d)}{\partial \Delta d} \Big|_{\Delta d=0} \Delta d \right\|_2 + \|\Delta A(d + \Delta d)\|_2 \right) \end{aligned} \tag{8.13}$$

where $\tilde{\lambda}_i$ is the eigenvalue of $A(d + \Delta d)$.

Assume that $\|\Delta A(d + \Delta d)\|_2$ is still smaller than S. Then the inequality (Equation 8.13) may be rewritten as

$$\begin{aligned} \mathrm{Re}(\lambda_i^0) - K \left(\left\| \frac{\partial A_0(d + \Delta d)}{\partial \Delta d} \Big|_{\Delta d=0} \Delta d \right\|_2 + S \right) &\le \mathrm{Re}(\tilde{\lambda}_i) \\ &\le \mathrm{Re}\left(\lambda_i^0\right) + K \left(\left\| \frac{\partial A_0(d + \Delta d)}{\partial \Delta d} \Big|_{\Delta d=0} \Delta d \right\|_2 + S \right) \end{aligned} \tag{8.14}$$

Then, the variation bounds of these design variables in the stability variable space S_s are determined. Limit variations of the design variables in the bounds $\{\Delta d_1 \in [-D_1, D_1], \ldots, \Delta d_m \in [-D_m, D_m]\}$ with D_i as the maximum values of Δd_i. When design variables vary within their bounds, the variation term $K \cdot (\|\frac{\partial A_0(d+\Delta d)}{\partial \Delta d}|_{\Delta d=0} \Delta d\|_2 + S)$ is smaller than all $|\mathrm{Re}(\lambda_i^0)|$ $(i = 1, \ldots, n)$ so that the real part of $\tilde{\lambda}_i$ is smaller than zero. As a result, these variation bounds will guarantee that the system is still stable. Such bounds can be obtained by solving the following feasible problem (P_3):

$$
\begin{aligned}
&\textit{Feasible } (P_3): \quad \textit{Find } (D_1, \ldots, D_m)\\
&\textit{to make}\\
&K \cdot \left(\left\| \frac{\partial A_0(d+\Delta d)}{\partial \Delta d} \Big|_{\Delta d=0} \Delta d \right\|_2 + S \right) < \min_i \left(\left| \mathrm{Re}\left(\lambda_i^0\right) \right| \right) \\
&\Delta d_j \in [-D_j, D_j] \ (i = 1, \ldots, n; j = 1, \ldots, m)\\
&h(d+\Delta d) = 0, l(d+\Delta d) \le 0\\
&d \in S_s
\end{aligned}
\tag{8.15}
$$

The inequalities $K \cdot (\| \frac{\partial A_0(d+\Delta d)}{\partial \Delta d} |_{\Delta d=0} \Delta d \|_2 + S) < \min_i (|\mathrm{Re}(\lambda_i^0)|)(i = 1, \ldots, n)$ mean that all system eigenvalues are smaller than zero, even if model uncertainty and variations of the design variables exist in the system.

Finally, it is important to judge whether the bounds $(D_1, \ldots, D_m)$ are larger than the desirable levels $(\varepsilon_1, \ldots, \varepsilon_m)$ specified by users according to their practical requirements. If $\varepsilon_1 \le D_1$ and $\ldots$ and $\varepsilon_{\mathrm{m}} \le D_m$, then these stability design variables d can be accepted. All these accepted stability design variables d and their variation bounds form a robust stability variable space S_{r}. When the design variables d vary within this robust stability variable space S_{r}, system stability will be maintained.

8.4 ROBUST EIGENVALUE DESIGN AND TOLERANCE DESIGN

8.4.1 Robust Eigenvalue Design

In this section, the robust eigenvalue design will be chosen from the robust stability parameter space S_r to minimize the variations of eigenvalues so that the system dynamic response will be less sensitive to both model uncertainty and variations of the design variables.

Equation 8.13 may be rewritten as

$$
\left| \tilde{\lambda}_i - \lambda_i^0 \right| \le K \cdot \left(\left\| \frac{\partial A_0(d+\Delta d)}{\partial \Delta d} \Big|_{\Delta d=0} \right\|_2 \cdot \|\Delta d\|_2 + \|\Delta A(d+\Delta d)\|_2 \right) \tag{8.16}
$$

Since $\|\Delta A(d+\Delta d)\|_2 \le S$ and $\|\Delta d\|_2 \le R$ which can be derived from the bounds of the design variables, from the inequality (Equation 8.16), if $K \cdot (\| \frac{\partial A_0(d+\Delta d)}{\partial \Delta d} |_{\Delta d=0} \|_2 \cdot R + S)$ is very small compared to $|\lambda_i^0|$, then

1. $\tilde{\lambda}_i$ is close to λ_i^0.
2. $\tilde{\lambda}_i$ is less sensitive to $\Delta \tilde{A}$.

Moreover, if $K \cdot (\|\frac{\partial A_0(d+\Delta d)}{\partial \Delta d}|_{\Delta d=0}\|_2 \cdot R + S) \ll |\lambda_i^0|$, then λ_i will be approximately equal to λ_i^0, which means that the eigenvalues of the practical system are less sensitive to both model uncertainty and variations of the design variables. As a result, the system will have a robust eigenvalue design. Thus, if $K \cdot (\|\frac{\partial A_0(d+\Delta d)}{\partial \Delta d}|_{\Delta d=0}\|_2 \cdot R + S)$ is minimized by selecting a set of suitable design variables d, then λ^0 will be close to λ, even if there exist model uncertainty and variations of the design variables. Such variables d for the robust eigenvalue design can be determined by solving the following optimization problem

$$\begin{aligned} &\min_d K \cdot \left(\left\| \frac{\partial A_0(d + \Delta d)}{\partial \Delta d} \right|_{\Delta d=0} \bigg\|_2 \cdot R + S \right) \\ &s.t.\ d \in S_r \end{aligned} \tag{8.17}$$

where $d \in S_r$ is the requirement of stability and feasibility. The robust design obtained from the optimization (Equation 8.17) can achieve both stability and robustness for the dynamic system despite uncertainties.

8.4.2 Tolerance Design

In order to ensure that the system has a satisfactory dynamic response, the eigenvalue variations of the obtained robust design should be limited in a desirable domain Y_r defined by users,

$$\|\Delta \lambda\|_2^2 \le Y_r^2 \tag{8.18}$$

This constraint (Equation 8.18) may be satisfied by designing the tolerance of the obtained robust design. Let the tolerance space S_t as $\{\Delta d_1 \in [-\delta d_1, \delta d_1], \ldots, \Delta d_m \in [-\delta d_m, \delta d_m]\}$, where $\delta d_1, \ldots, \delta d_m$ is the largest variation of the design variables under the condition (Equation 8.18). The principle of the tolerance design is to maximize the tolerance space under the performance constraint Y_r while maintaining the stability and robustness of the system.

From (Equation 8.16) and $\|\Delta A(d + \Delta d)\|_2 \le S$, the inequality (Equation 8.18) can be rewritten as

$$\begin{aligned} \|\Delta \lambda\|_2^2 &= \sum_{i=1}^{n} \left(|\tilde{\lambda}_i - \lambda_i^0|\right)^2 \\ &\le n \left(K \cdot \left(\left\| \frac{\partial A_0(d + \Delta d)}{\partial \Delta d} \right|_{\Delta d=0} \delta d \bigg\|_2 + S \right) \right)^2 \le Y_r^2 \end{aligned} \tag{8.19}$$

The tolerance of the design variables can be obtained by solving the following tolerance optimization:

$$
\begin{aligned}
&\max_{\delta d_1,\ldots,\,\delta d_2} \prod_{i=1}^{m} \delta d_i \\
&st. \\
&\sqrt{n}\left(K \cdot \left(\left\| \frac{\partial A_0(d+\Delta d)}{\partial \Delta d} \right|_{\Delta d=0} \delta d \right\|_2 + S\right)\right) \le Y_r \\
&\gamma_{\mathrm{i}} \le \delta d_i \le D_i \; i = 1, \ldots, m
\end{aligned}
\tag{8.20}
$$

where γ is the lower bound of the tolerance δd and is specified by users according to their practical requirements; the constraint $\delta d_i \le D_i$ represents that the tolerance is constrained in the robust stability variable space S_r, which will guarantee the stability and robustness of the system; and the constraint $\sqrt{n}(K \cdot (\| \frac{\partial A_0(d+\Delta d)}{\partial \Delta d} |_{\Delta d=0} \delta d \|_2 + S)) \le Y_r$ means that the eigenvalue variations are limited within a desirable range. Thus, the system is still stable and robust, and produces a satisfactory dynamic response when the robust design varies within its tolerance space.

8.4.3 Design Procedure

The design procedure for the newly presented robust approach is summarized as follows:

Step 1: Find the nominal stability space S_n for design variables by solving the feasible problem (P_1). These nominal design variables guarantee the nominal system stability.

Step 2: Find the stability space S_s for design variables within the nominal stability space S_n by solving the feasible problem (P_2). When design variables vary within this stability space S_{s}, stability is also maintained even if model uncertainty exists.

Step 3: Find the robust stability space S_r for design variables within the stability space S_s by solving the feasible problem (P_3). When design variables vary within this robust stability space S_r, stability is still maintained, even if there exist model uncertainty and variations of the design variables.

Step 4: Find the robust eigenvalue design within the robust stability space S_r by solving the optimization problem (Equation 8.17). This robust design minimizes variations of eigenvalues so that the system dynamic response will be less sensitive to uncertainties.

Step 5: Find the tolerance of the obtained robust design by solving the optimization problem (Equation 8.20). If the robust design varies within its tolerance space, this design will feature a satisfactory dynamic response.

8.5 CASE STUDY

Consider the laval rotor systems as Example 1.3 given in Section 1.1.2 of Chapter 1. The state-space equation for the rotor system is

$$\dot{x} = Ax \tag{8.21}$$

with the Jacobian matrix $A = A_0 + \Delta A$ and the nominal matrix $A_0 = \begin{bmatrix} -M^{-1}D & -M^{-1}R \\ I_{4\times 4} & 0_{4\times 4} \end{bmatrix}$.

The parameter values are shown in Table 6.1, and the model parameter p can vary randomly in [0, 1] and is uncontrollable. The objective is to select the design variables m and m_b from $m \in [1\text{kg}, 4\text{kg}]$ and $m_b \in [1\text{kg}, 5\text{kg}]$ to make the system stable and minimize the effect of model uncertainty and variations of the design variables on the eigenvalues.

8.5.1 Design of the Nominal Stability Space

A nominal stability space S_n for design variables is figured out by solving the feasible problem (P_1). This space S_n is shown in Figure 8.1, where the blank space stands for unstable design and the shadow space stands for stable design.

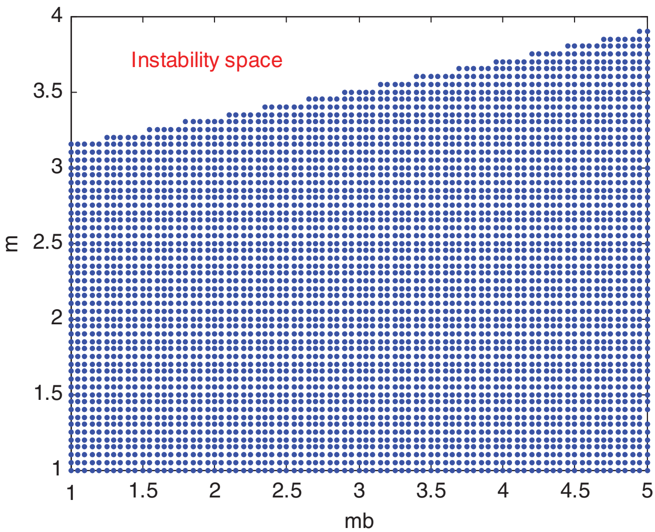


FIGURE 8.1 Nominal stability space S_n for design variables (m, m_b)

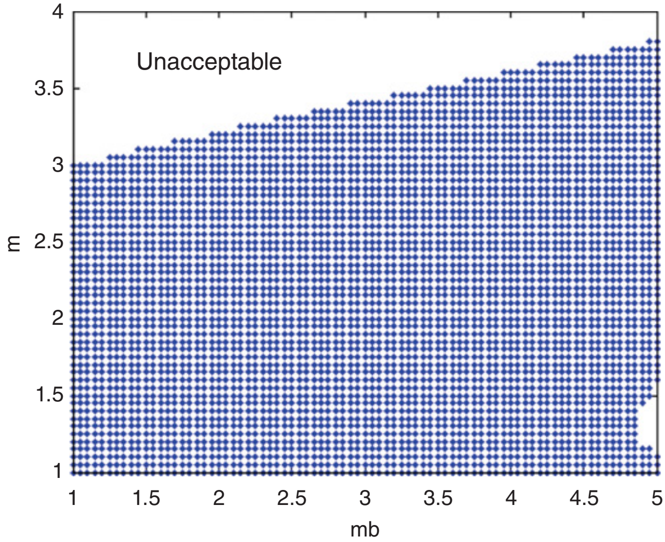


FIGURE 8.2 Stability space S_s for design variables (m, m_b)

8.5.2 Design of the Stability Space

Then, the stability space S_s for design variables is derived by solving the feasible problem (P_2) within the nominal stability variable space S_n. From the estimation ofΔA, the upper bound S of $\|\Delta A\|_2$ is equal to 0.01. This space S_s is shown in Figure 8.2, where the blank spaces stand for the failure design space and the shadow space stands for the acceptable design space.

8.5.3 Design of the Robust Stability Space

Furthermore, the robust stability space S_r for design variables is determined by solving the feasible problem (P_3) within the stability space S_s with the desirable level $\varepsilon_1 = \varepsilon_2 = 0.2$ according to practical requirements. This space S_r is shown in Figure 8.3, where the blank space stands for the failure design space and the shadow space stands for the acceptable design space.

8.5.4 Robust Eigenvalue Design

The robust design variables gained from Equation 8.17 are $m = 3.5$ kg and $m_b = 5$ kg. The relative approximation error ζ between the exact eigenvalue λ_i and the nominal eigenvalue λ_i^0 is defined as below

$$\zeta = \frac{|\lambda_i - \lambda_i^0|}{|\lambda_i^0|} \tag{8.22}$$

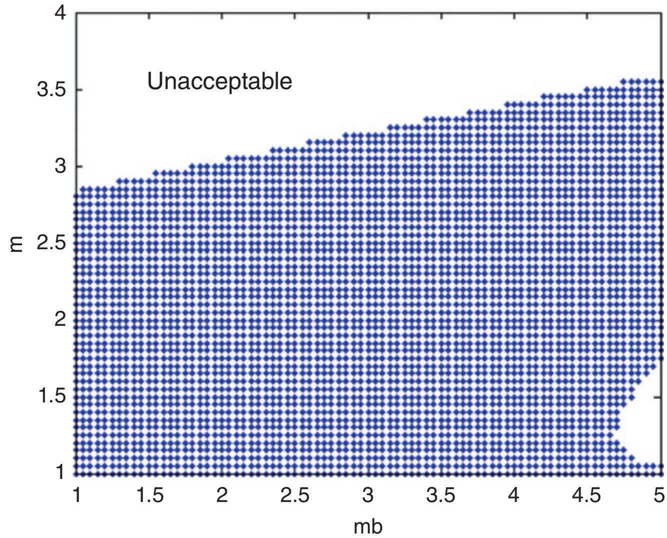


FIGURE 8.3 Robust stability space S_r for design variables (m, m_b)

The exact eigenvalues and the relative approximate error under different parameter p are shown in Table 8.1, where the real parts of all eigenvalues are negative and all relative approximate errors are very small. So the robust design ($m = 3.5$ kg and $m_b = 5$ kg) not only achieves robustness but also guarantees the system to be stable even if the model uncertainty exists.

8.5.5 Tolerance Design

Assume that the performance constraint is defined as $Y_r = 0.01$. The largest tolerance space of the obtained robust design ($m = 3.5$ kg and $m_b = 5$ kg) will be solved from Equation 8.20. The tolerance space and the maximum variation of the design variables are shown in Table 8.2. These tolerances are easy to realize because the tolerance 0.0031 kg is simple to manufacture and measure using a common manufacturing process and a common balance.

8.5.6 Design Verification

Let Δm and Δm_b vary randomly in $[-0.01, 0.01]$ and the parameter p also varies randomly in [0, 1]. A total of 1000 samples are taken to check the presented robust design.

For more effective verification, comparison is carried out utilizing two design methods: (a) the present robust design method, and (b) the previous robust design method in Chapter 6 that considers the effect of variations of the design variables

TABLE 8.1 Stability-Based Robust Performance (m = 3.5 kg, m_b = 5 kg)

p	λ_1, λ_2	$\zeta_{1,2}(\%)$	λ_3, λ_4	$\zeta_{3,4}(\%)$	λ_5, λ_6	$\zeta_{5,6}(\%)$	λ_7, λ_8	$\zeta_{7,8}(\%)$
0	−0.0571 ± 4.7536i	0.003	−1.5515 ± 4.7679i	0.003	−1.9897 ± 10.2494i	0.0034	−0.3150 ± 10.2636i	0.0035
0.2	−0.0571 ± 4.7537i	0.0039	−1.5515 ± 4.7679i	0.0040	−1.9897 ± 10.2494i	0.0034	−0.3150 ± 10.2636i	0.0035
0.4	−0.0570 ± 4.7537i	0.0048	−1.5514 ± 4.7679i	0.0048	−1.9897 ± 10.2494i	0.0033	−0.3150 ± 10.2636i	0.0034
0.6	−0.0570 ± 4.7537i	0.0056	−1.5514 ± 4.7679i	0.0056	−1.9897 ± 10.2494i	0.0032	−0.3150 ± 10.2636i	0.0032
0.8	−0.0569 ± 4.7537i	0.0063	−1.5513 ± 4.7679i	0.0063	−1.9897 ± 10.2494i	0.0029	−0.3151 ± 10.2636i	0.0029
1	−0.0569 ± 4.7537i	0.0068	−1.5513 ± 4.7679i	0.0068	−1.9898 ± 10.2494i	0.0026	−0.3151 ± 10.2636i	0.0026

TABLE 8.2 Performance-Based Tolerance

Y_r	Largest Tolerance Space	Tolerance δ^m	Tolerance δ^{m_b}
0.01	9.61×10^{-4}	0.0031	0.0031

TABLE 8.3 Comparison Under the Model Uncertainty

Performance Variation W	Previous Method	Present Method
Mean μ_W	0.0062	0.003
Variance σ_W	3.1523e-005	9.5256e-006

only without considering the model uncertainty. The solution of the second method is $m = 2$ kg and $m_b = 4.6$ kg.

The performance variation $W$is defined as

$$W = \|\Delta\lambda\|_2^2$$

The performance index E is defined as

$$E = W_T - W_p$$

where W_T and W_p are the performance variation W gained by the previous robust design method and the present robust design method, respectively. The performance index E will tell their difference. Only if the percent of $E > 0$ is larger than 50%, the robustness of the present method will be better than the previous one. Otherwise, the previous robust design is better.

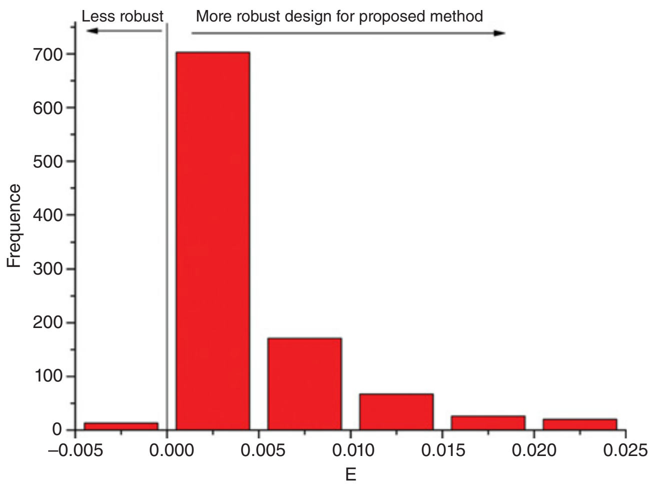


FIGURE 8.4 Comparison under the model uncertainty

From Table 8.3, we can see that both the mean and variance of the performance variation W gained by the present robust design method are smaller than those by the previous design method. From Figure 8.4, it is clear that the present approach has about a 98.7% (for $E > 0$) chance to achieve a better design than the previous one. So the present robust design method is more robust than the previous robust design method, because the present robust design method considers model uncertainty, whereas the previous robust design method does not.

8.6 SUMMARY

A novel robust design method is presented to ensure stability and robustness of the system under model uncertainty. In this design method, stability design can effectively guarantee system stability even if model uncertainty exists. Moreover, robust eigenvalue design is able to minimize the effect of uncertainties on system eigenvalues, so that desirable dynamic performance can be achieved. Given the performance constraints, tolerance design can further maximize the working space for design variables while maintaining the stability and robustness unchanged. The selected case study also demonstrates the effectiveness of this method.

PART IV

INTEGRATION OF DESIGN AND CONTROL

CHAPTER 9

DESIGN-FOR-CONTROL-BASED INTEGRATION

A design-for-control-based integration method is presented for nonlinear system with hybrid discrete/continuous variables in this chapter. The system is first designed to have approximately linear dynamics. This designed dynamics can be easily handled by a simple controller. Under this framework, robust design and control are integrated to achieve the desired dynamic performance under parameter uncertainty.

9.1 INTRODUCTION

Most of the manufacturing processes consist of both discrete and continuous variables. The hybrid discrete/continuous system is more complex than the traditional system. System design is usually a discrete function that sets design variables offline, while process control is a continuous function that keeps adjusting the process variables online. With the increasing production accuracy, it would be difficult to obtain high performance manufacturing by sequential approach at which system design and control are separated. This is because design constraints are not considered in continuous control development and controllability is not taken into account at the stage of process design. If system design and control can be integrated effectively, it would greatly enhance the overall performance of manufacturing process.

In control engineering, many nonlinear control techniques are developed to control the nonlinear process. However, these techniques must satisfy many stringent

System Design and Control Integration for Advanced Manufacturing, First Edition. Han-Xiong Li and XinJiang Lu.

conditions (Sun and Hoo, 2000), which often cause them impractical and difficult to realize. Alternatively, many nonlinear processes are often replaced by linear models since they can be described very well by linear models in the area of interest (Sun and Hoo, 2000). Then this kind of nonlinear processes can be controlled very well by linear control methods. In these alternative approaches, the extent of a system's nonlinearity needs to be examined first in order to make sure that the linear system analysis and controller synthesis methods are adequate to use. If system nonlinearity is strong, it should be minimized first through discrete process design. Inappropriate selection of design variables could make the system more nonlinear that would be difficult to the subsequent continuous control. On the other hand, proper selection of design variables could make the system more linear for an easy control later. Thus, a key problem is to find these suitable design variables for reducing nonlinearity and complexity of the system. Although existing integration approaches considered the dynamic performance in design and control simultaneously, they paid less attention to reduction of system nonlinearity and complexity at the stage of process design. This may cause the designed process difficult to control or the developed controller difficult to realize. Moreover, robustness of the system depends on not only system control but also system design. Although there are many works reported to achieve robust pole assignment at the stage of control, there is still no similar work for the system design, let alone the integration of design and control. An effective integration approach is very necessary to develop for the robust pole assignment in both process design and process control.

An integrated design and control method is presented in this chapter for the nonlinear system with hybrid discrete/continuous variables. First, in order to simplify controller design, a design-for-control approach is presented to make the complex system to have a well linear approximation and controllable property. Then, the robust design and control is integrated to guarantee system stability as well as the robust pole placement. Finally, effectiveness of this integration method is demonstrated first on the numerical simulation and then on the real manufacturing process.

9.2 INTEGRATION PROBLEM

Consider a nonlinear hybrid system:

$$\dot{x}(t) = f(x(t), d) + Bu(t) \tag{9.1}$$

where $x \in R^n$ represents state vector, $f(\cdot)$ is nonlinear function, u is the continuous control variable that will be adjusted online, B is the constant matrix, and $d \in R^m$ is the discrete parameter with the nominal value needs to design offline. It is assumed there is uncontrollable variation around the nominal value of the design parameter.

Generally, design parameter has a significant impact on process dynamics. However, the traditional process design usually considers only the static performance of the system and leaves the transient dynamics for the controller to handle. The system designed may exhibit undesired dynamic behavior. As an example, in Figure 9.1,

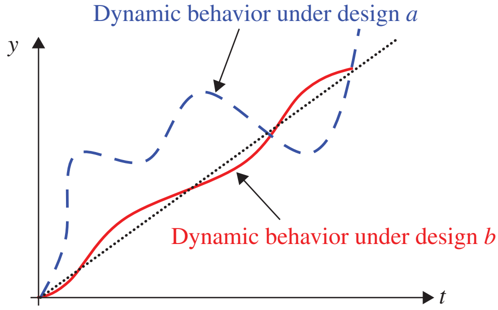


FIGURE 9.1 Dynamic behavior under different designs

the design a demonstrates more nonlinear and complex behaviors than the design b. This complex nonlinear system renders difficulties for control engineers to develop a suitable control method. Even if the controller can be found for this kind of system, it may be complex and not easy to implement. For a satisfactory performance using a relatively simple control algorithm, the nonlinearity and complexity of the system should be minimized first at the stage of process design. Of course, it would be even better if the system designed can be approximated well by a linear model. Therefore, an effective method should be proposed to design the system to have an approximately linear property without loss of other design performances.

From the point of view of production, the system should have a desirable robust dynamic performance to obtain consistent production. From the point of view of control, the system should be controllable and have an approximately linear behavior. From the point of view of economics, economic criteria must be desired so that the system designed is reasonable and realizable. Thus, the integration of design and control problem can be described as

$$\begin{aligned} &\min_{d,u} (Dynamic\ robust\ performance) \\ &\quad s.t.\ \ Feasible\ problem \end{aligned} \tag{9.2a}$$

with

$$\begin{aligned} &\text{Feasible problem} : find\ d\ for\ linear\ approximation/controllability \\ &\qquad s.t.\ \ h(x(t), d, u(t)) \le 0,\ \ d_L \le d \le d_U, u \in U \end{aligned} \tag{9.2b}$$

where $h(.)$ is the constraint function, including economic constraints and other constraints in manufacturing industry, such as equipment size and weight, d_L and d_U are lower and upper bound of design variable d, and u is the control variable defined on the space U.

9.3 DESIGN-FOR-CONTROL-BASED INTEGRATION METHODOLOGY

It is well known that the performance of any system depends on not only the external control but also its own design. On the one hand, since design variable has a significant impact on process dynamics, the system design becomes critical to the transient response of any dynamic system. A good design has many advantages for control. It can share some working load of the controller, such that a simple controller could be used without sacrificing the overall performance. On the other hand, the system dynamic performance depends more on the controller design. Thus, an integrated design and control method is presented in Figure 9.2 to achieve the desirable design/control performance. First, a design-for-control approach is used to design the system to have a well linear approximation and controllable property. This advantage may simplify the controller design. Then, the process design and a common feedback control are integrated to simultaneously consider the stability and the robust pole placement. A robust design method is employed to obtain design variables and control parameters for this integration. This integration method has the following characteristics:

- System task will be shared by discrete design and continuous control: the easy-control property is considered at the stage of process design, upon which a simple controller will be designed latter for a robust global performance.
- The robust pole placement can be considered simultaneously in both process design and controller synthesis.

9.3.1 Design for Control

In order to achieve a satisfactory dynamic performance using a relatively simple and easy-realization controller, a design-for-control method is presented as shown in Figure 9.3 to reduce system nonlinearity and guarantee its controllability through the process design. First, a feasible design is employed to obtain all acceptable design

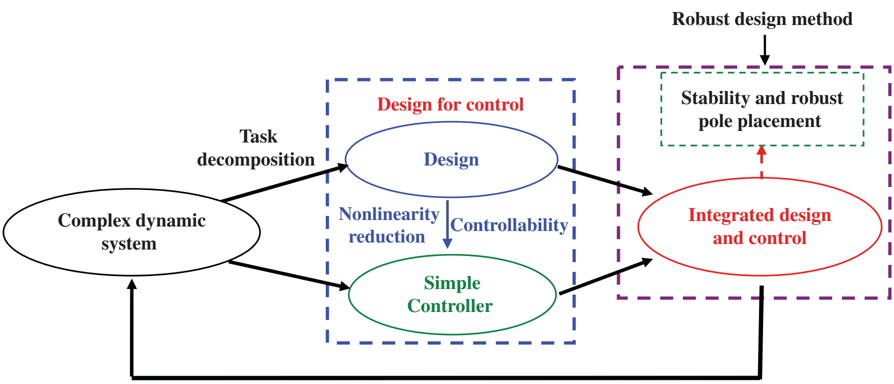


FIGURE 9.2 Integrated design and control methodology

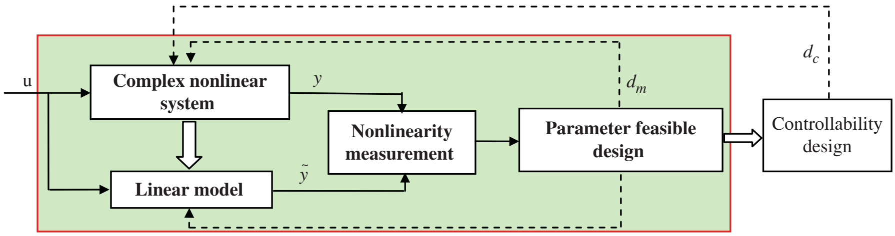


FIGURE 9.3 Design-for-control methodology

variables for a desirable linear approximation. Then, a controllability design is used to guarantee desirable positions of the system poles.

9.3.1.1 *Feasible Design for Linear Approximation* Nonlinearity measurement is to estimate the approximation degree between the nonlinear system and the most suitable linear system chosen from the linear operator space under the given input space. A common nonlinearity measurement (Schweickhardt and Allgower, 2004, 2007; Lu et al., 2011) is described as:

$$\phi(d) = \sup_{u \in U} \frac{\|Nu - Lu\|}{\|Nu\|} \tag{9.3}$$

where $N : u \to x$ is the model of the original system (Equation 9.1), and $L : u \to x$ denotes a linear model $\dot{x}(t) = \bar{A}x(t) + Bu(t)$ that is obtained by the linearization method at the operating point. Both the model nonlinearity N and its linear approximation L are functions of design variable d. The measurement $\phi(d)$ is a number between zero and one, with its value close to zero indicating the existence of a linear approximation to the system and its value close to one indicating a highly nonlinear system.

Based on this nonlinearity measurement, in order to make the system to have a well linear approximation, the measurement $\phi(d)$ should be smaller than a desirable value ϕ_0 set by users. Thus, a set of suitable design variables should be chosen to satisfy this condition.

(P_1): Find (d)

To make

$$\phi(d) \le \phi_0,\ h(x(t), d, u(t)) \le 0, d_L \le d \le d_U \tag{9.4}$$

All solutions of the feasible problem (P_1) form the suitable space S_m for design variables, in which any design variable will make the system to have a well linear approximation.

9.3.1.2 *Controllability Design* Then, according to the dynamic theory, the condition that the system poles can be placed at arbitrary positions is that the system can be fully controllable. Thus, the design variable should be chosen to make the pair $(\bar{A}, B)$ fully controllable.

(P_2): Find (d)

To make

$$(\bar{A}, B) \text{ is fully controllable} \tag{9.5}$$

$$h(x(t), d, k) \leq 0, d \in S_m$$

where $d \in S_m$ is to guarantee a well linear approximation. All solutions of the controllable problem (P_2) form a space S_c for controllable parameters, in which any design variable will make the system to have a well linear dynamic behavior and controllable property.

9.3.2 Control Development

After the design-for-control is carried out, the systems in the space S_c for suitable design variables have a well linear dynamic behavior and controllable property. This can make their dynamic performance easier to obtain using a simple controller. Thus, a relatively simple state-feedback controller is adopted to control these systems since it is a common control method widely used in manufacturing. This state-feedback control rule will be described as

$$u(t) = kx \tag{9.6}$$

where k is the gain matrix. Thus, the total system model can be rewritten as

$$\dot{x} = A(d, k)x \tag{9.7}$$

with $A = \bar{A} + Bk$.

The design variable d and the control gain matrix k in the parametric model (Equation 9.7) are required to be optimized simultaneously for the global robust dynamic performance.

9.3.3 Integration Optimization for Robust Pole Assignment

It is well known that the positions and robustness of the system eigenvalues are extremely important to the dynamic performance. This section will integrate the process design and control to consider them in order to guarantee the system stability and achieve its dynamic robustness.

The objective of this integration of design and control is to choose the suitable design variable d and control gain k to make the eigenvalues of the system to be in the desirable locations and less sensitive to uncertainty. For this objective, two sequential designs are developed:

1. ***Pole assignment of the nominal system without considering parameter variations.*** It is to find a nominal parameter space S_n, consisting of these design and control gains that can place eigenvalues of the nominal matrix $A(d, k)$ to be in desirable locations.
2. ***Stability design and robust pole assignment.*** It is to choose robust design variable and control gain from S_n to guarantee the system stability and make all eigenvalues less sensitive to uncertainty.

9.3.3.1 Pole Assignment of the Nominal System This design is to realize the pole placement of the nominal system. The desirable self-conjugate eigenvalue λ_i^r and its set Λ_0 are given as below

$$\Lambda_0 = diag\ \left\{\lambda_1^r, \dots, \lambda_n^r\right\} \tag{9.8}$$

Then, design variable and control gain are chosen to make eigenvalues $\lambda(d, k)$ of the nominal matrix $A(d, k)$ to be the desirable eigenvalues Λ_0 and satisfy other constraints

(P_3): Find (d, k)

To make

$$\begin{aligned} \Lambda_0 &= \lambda(d, k), \\ h(x(t), d, k) &\leq 0,\ \ d \in S_c \end{aligned} \tag{9.9}$$

where $d \in S_c$ can guarantee the system poles to be arbitrarily placed and make the system designed to have a well linear approximation. All solutions of the nominal pole placement problem (P_3) form a nominal parameter space S_n, in which every design variable will make the nominal system to have a desirable dynamic performance.

9.3.3.2 Stability Design and Robust Pole Assignment This design is to choose design variable and control gain from the nominal parameter space S_n to have system stability and robust pole assignment under uncertainty.

1. Stability design

According to the orthogonal theory of eigenvector, the system eigenvalue λ_i $(i = 1, \dots, n)$ is shown below with the detailed derivation in Chapter 6

$$\lambda_i(d + \Delta d, k) \approx \lambda_i(d, k) + \left(v_i^T \frac{\partial A}{\partial \Delta d} \mu_i\right)\Big|_d \Delta d^T \tag{9.10}$$

where μ_i and v_i are the right eigenvector and the left eigenvector for the eigenvalue λ_i, respectively.

From the pole assignment of the nominal system, it is clear that

$$\lambda_i(d,k) = \lambda_i^r(d,k) \tag{9.11}$$

where $\lambda_i(d,k)$ and $\lambda_i^r(d,k)$ are the ith element of $\lambda(d,k)$ and Λ_0, respectively.

Inserting Equations 9.11 into 9.10, we have

$$\lambda_i(d+\Delta d,k) \approx \lambda_i^r(d,k) + \left(v_i^T \frac{\partial A}{\partial \Delta d}\mu_i\right)\bigg|_d \Delta d^T \tag{9.12}$$

In order to have the system stability under uncertainty, the real part of $\lambda_i(d+\Delta d,k)$ should be smaller than zero. Thus, we have the following theorem:

Theorem 9.1 Consider the system (Equation 9.1) with variation of design variable. Under the design-for-control (Equations 9.3, 9.4, and 9.5) and given the nominal parameter space S_n and Equation 9.11, if $|\mathrm{Re}((v_i^T \frac{\partial A}{\partial \Delta d}\mu_i)|_d \Delta d^T)| < |\mathrm{Re}(\lambda_i^\gamma(d,k))|$ is satisfied when Δd arbitrarily varies in its constraint space, where $\mathrm{Re}(\cdot)$ means the real part of $(\cdot)$, then all eigenvalues of A have negative real parts (A is a Hurwitz matrix) and the system (Equation 9.1) is asymptotically stable even if uncertainty exists.

Proof: Since all design variables are limited within S_n, $\mathrm{Re}(\lambda_i^r(d,k))$ is smaller than zero. From Equation 9.12, if $|\mathrm{Re}((v_i^T \frac{\partial A}{\partial \Delta d}\mu_i)|_d \Delta d^T)| < |\mathrm{Re}(\lambda_i^\gamma(d,k))|$, the real part of $\lambda_i(d+\Delta d,k)$ will be smaller than zero. Therefore, the system (9.1) is asymptotically stable. ■

Thus, the suitable design variables and control gains should be found to satisfy this stability condition, which forms the following stability problem (P_4):

$$\begin{aligned} (P_4):\quad & \textit{Find } (d,k) \\ & \textit{to make} \\ & \left|\mathrm{Re}\left(\left(v_i^T \frac{\partial A}{\partial \Delta d}\mu_i\right)\bigg|_d \Delta d^T\right)\right| < \left|\mathrm{Re}\left(\lambda_i^r(d,k)\right)\right| \\ & \Delta d_L \le \Delta d \le \Delta d_U,\quad (d,k)\in S_n \end{aligned} \tag{9.13}$$

where Δd_L and Δd_U are a lower and upper bound of perturbation of design variable, $(d,k) \in S_n$ can guarantee the eigenvalues of the nominal system placed on the desirable poles. All possible solutions of the stability problem (P_4) form a stability variable space S_s, in which every parameter will guarantee system stability even if uncertainty exists.

2. Robust pole assignment

From Equation 9.10, the eigenvalue variation $\Delta\lambda_i$ with respect to perturbation of design variable can be expressed as

$$\Delta\lambda = J\Delta d \tag{9.14}$$

where

$$\Delta\lambda_i = \lambda_i(d + \Delta d, k) - \lambda_i^r(d, k), \Delta\lambda = [\Delta\lambda_1 \ \dots \ \Delta\lambda_n]^T,$$

$$J = \begin{bmatrix} \left(v_1^T \dfrac{\partial A}{\partial \Delta d_1}\mu_1\right)\Big|_d & \cdots & \left(v_1^T \dfrac{\partial A}{\partial \Delta d_m}\mu_1\right)\Big|_d \\ \vdots & \ddots & \vdots \\ \left(v_n^T \dfrac{\partial A}{\partial \Delta d_1}\mu_n\right)\Big|_d & \cdots & \left(v_n^T \dfrac{\partial A}{\partial \Delta d_m}\mu_n\right)\Big|_d \end{bmatrix}$$

As presentation in Chapter 6, performance will be directly related to design variable and control gain as follows:

$$\|\mathrm{Re}(\Delta\lambda)\|_2^2 + \|\mathrm{Im}(\Delta\lambda)\|_2^2 = \sum_{i=1}^{m} \sigma_i y_i^2 \tag{9.15}$$

with $[y_1, \dots, y_m]^T = V^T \Delta d$.
where $\mathrm{Im}(\cdot)$ means the imaginary part of $(\cdot)$, σ_i represents the singular value of $\mathrm{Re}(J)^T\ \mathrm{Re}(J) + \mathrm{Im}(J)^T\ \mathrm{Im}(J)$ and is a function of design variable and control gain, and the corresponding orthogonal eigenvector is denoted as V_i, which is one element of $V = [\,V_1 \ \cdots \ V_m\,]$.

According to the Euclidean norm method, only if the maximal singular value $\sigma_{\max}$ is minimized, the performance $\Delta\lambda$ will be less sensitive to uncertainty Δd. Thus, in order to minimize $\Delta\lambda$, design variable and control gain should be chosen to minimize the maximal singular value $\sigma_{\max}$ as follows.

$$\begin{aligned} &\min_{d,k}(\sigma_{\max}) \\ &s.t.\quad \|\mathrm{Re}(\Delta\lambda_d)\|_2^2 + \|\mathrm{Im}(\Delta\lambda_d)\|_2^2 = \sum_{i=1}^{m} \sigma_i y_t^2 \\ &(d, k) \in S_s \end{aligned} \tag{9.16}$$

where the condition $(d, k) \in S_s$ can guarantee system stability under uncertainty. The solution of Equation 9.16 can minimize the influence of uncertainty on system eigenvalues. This means that it can effectively obtain the robust eigenvalue placement under uncertainty.

9.3.4 Integration Procedure

This integrated design and control methodology is summarized in Figure 9.4. First, the system is designed to have a well linear approximation and controllable property, for which a relatively simple controller can be applied. Then, the system design

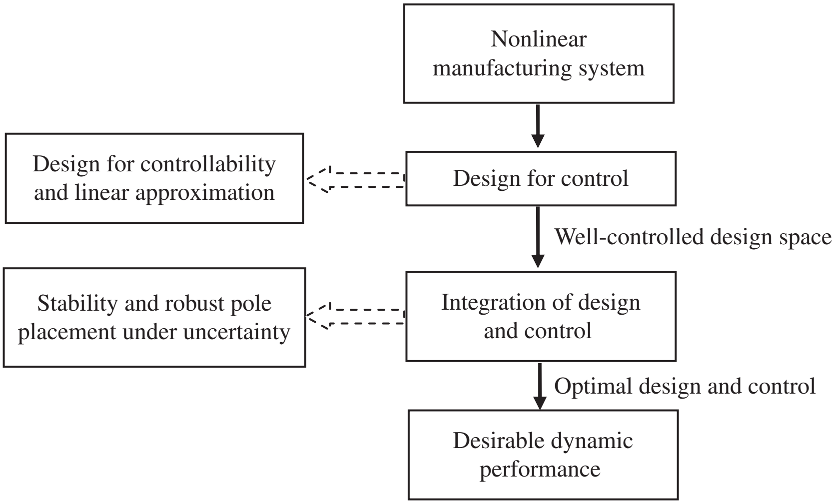


FIGURE 9.4 Design and control integration

and control design are integrated to guarantee the system stability and achieve the desirable robust performance under uncertainty. This integration method can achieve the robustness of the nonlinear system under uncertainty even if a relatively simple controller is used.

9.4 CASE STUDY

Example 9.1: Consider a design and control problem of a nonlinear system:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} (d^2-2)(x_1(t))^3 - x_1(t) + x_2(t) + 0.1x_1^2(t)x_2(t) \\ (-\sin(d)-1)x_2(t) + 0.05x_1(t)x_2(t) \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u \tag{9.17}$$

The design variable is d with the nominal value chosen from the design space $d \in [1, 2]$. There is uncontrollable variation Δd around the nominal value due to manufacturing and operating errors. To make the system well controlled, a set of suitable values should be found for d to have a well linear approximation under constraints. Then the control variable u could be determined optimally for a global robust dynamic performance. ■

9.4.1 Design for Control

In this design, the desirable value ϕ_0 in Equation 9.4 is set as 0.05, which is close to zero and thus this value can guarantee a well linear approximation to the original nonlinear system. The nonlinearity measurement calculated from Equation 9.3 is

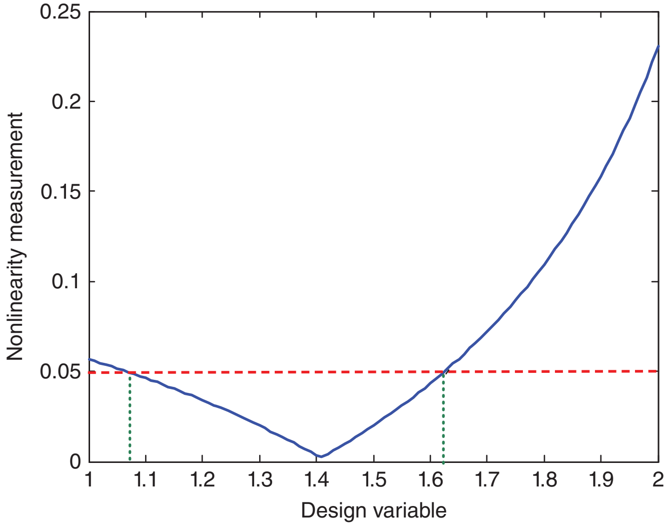


FIGURE 9.5 Nonlinearity measurement of design variable d

shown in Figure 9.5, from which it is clear that the suitable design space S_m is $d \in [1.07, 1.63]$. Then, the approximation performance between the nonlinear system and the linear model under $d = 1.41$ is shown in Figure 9.6, where it is clear that the linear model can approximate the nonlinear system well under this design.

9.4.2 Robust Pole Assignment

Suppose that a good dynamic response requires the desired poles to be on -1 and -2. In order to achieve this pole placement, the nominal design is conducted first to assign the poles of the nominal system to be the desired locations. Given the desired poles in the design, the nominal parameter space S_n is calculated next in Equation 9.9. Then, the robust dynamic design is obtained from this nominal parameter space S_n by solving the optimization problem Equation 9.16. The solution obtained are $d = 1.57$ and $K = [0 \;\; 0.317 \times 10^{-6}]$, and will be verified in Figure 9.7. Since the corresponding control gain K is already determined once the desired poles are given, so only the design variable is demonstrated in the figure. Obviously, the desired poles of the nominal system are -1 and -1.999999. The proposed design has achieved the goal.

9.4.3 Design Verification

The response of the system (x_1 and x_2) under the robust design can be considered as the nominal performance and shown in Figure 9.8. It is clear that the system is stable under nominal parameters derived from the proposed robust design.

Then, let Δd randomly varies in $(-0.05d^*, 0.05d^*)$where d^* is the robust design, and 1000 samples are used to evaluate the effectiveness of the presented method.

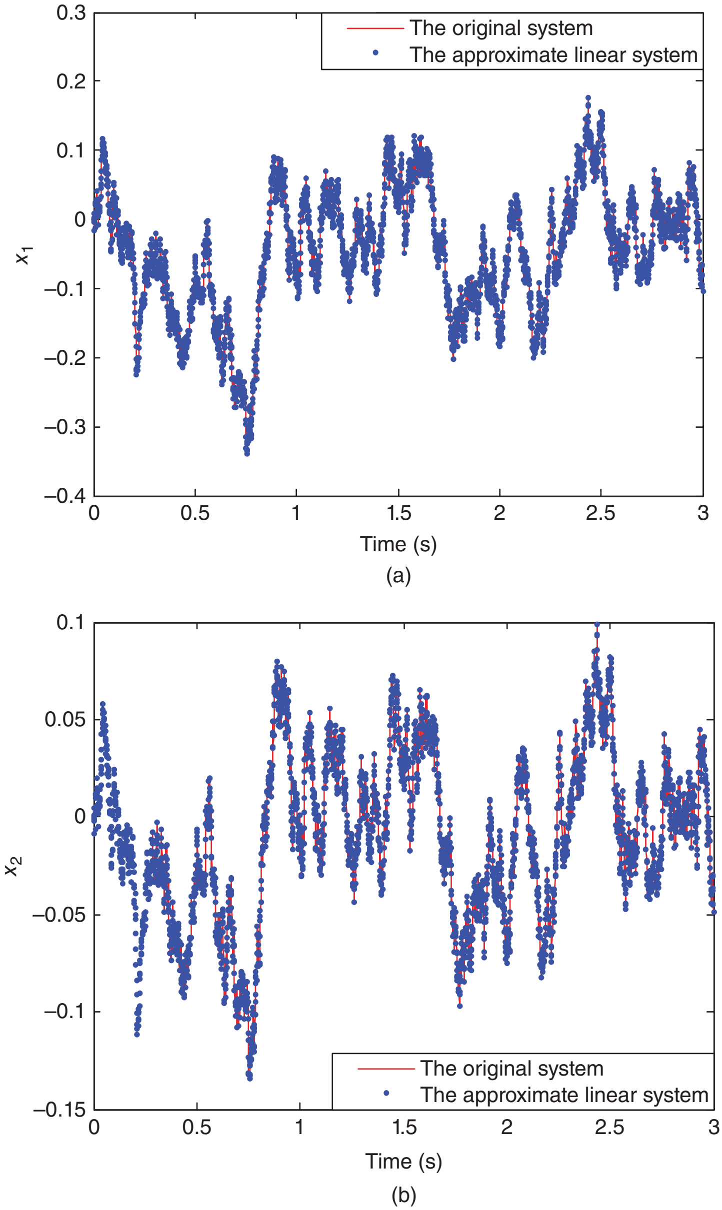


FIGURE 9.6 Approximation performance between the original system and the linear model: (a) state x_1; (b) state x_2

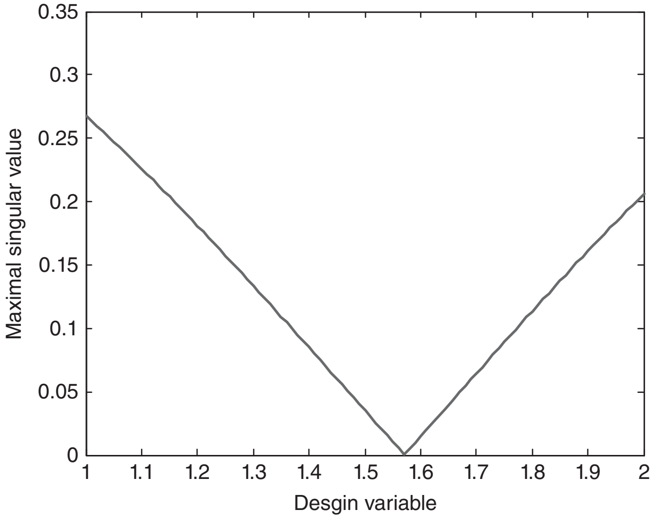


FIGURE 9.7 Maximal singular value under different design variable

Define the performance variations E_1 and E_2 as

$$E_{1,j}(t) = x_1(t) \text{ under } d^* + \Delta d_j - x_1(t) \text{ under } d^* \tag{9.18}$$

$$E_{2,j}(t) = x_2(t) \text{ under } d^* + \Delta d_j - x_2(t) \text{ under } d^* \tag{9.19}$$

where j is the jth sample of 1000 samples.

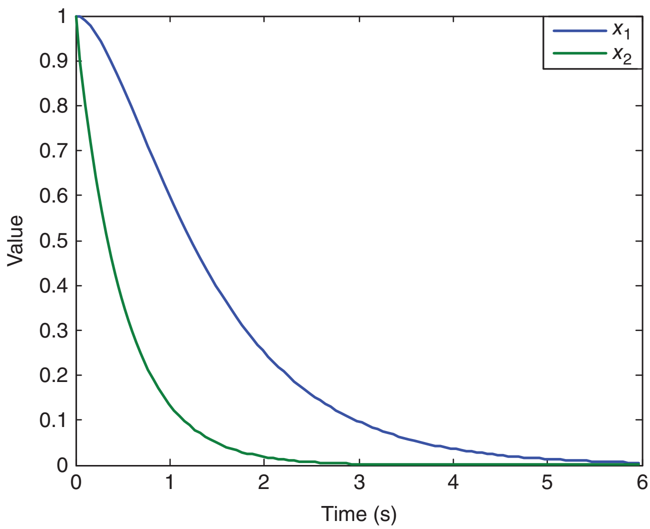
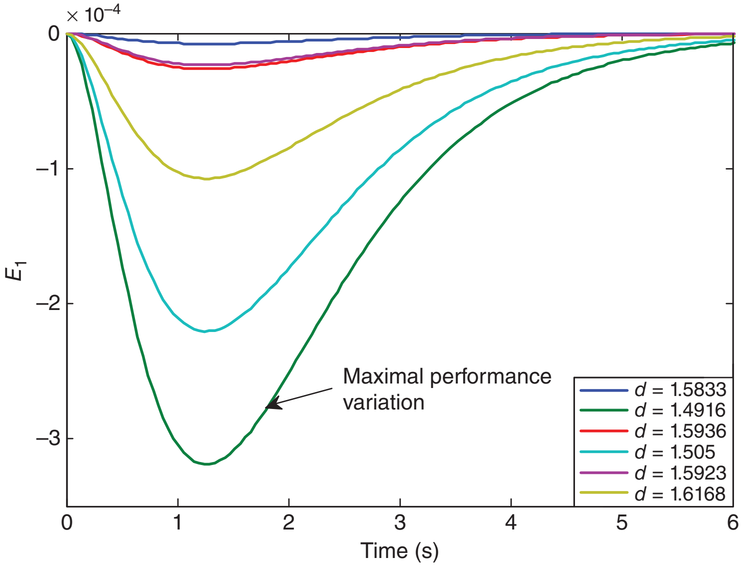
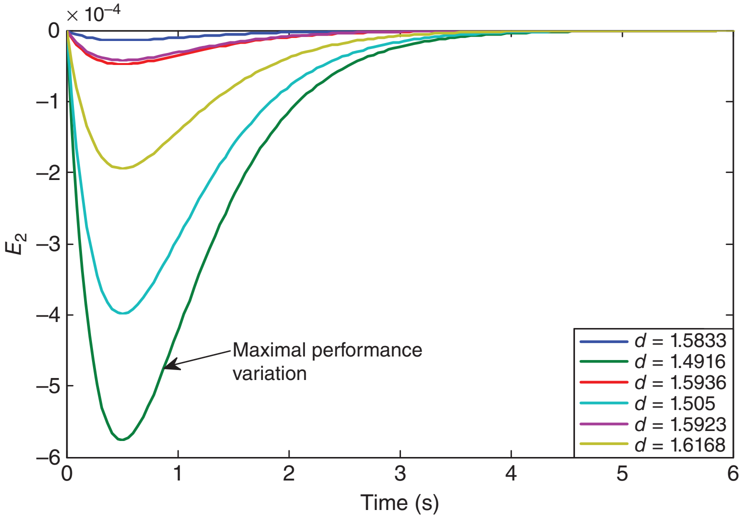


FIGURE 9.8 Nominal performance of the system under the robust design

All performance variations E_1 and E_2 from 1000 samples are calculated with the typical samples shown in Figure 9.9. It is clear that performance variations of the system are very small and can be ignored compared to the nominal performance of the system in Figure 9.8. Thus, the presented approach is very robust to parameter variations.



(a)



(b)

FIGURE 9.9 Performance variation caused by parameter variation Δd: (a) E_1; (b) E_2

Finally, the robust design is compared to other designs in the design space. Define the maximal root square mean error (RSME) as

$$\mathrm{RSME}(t,j) = \sqrt{\sum_{i=1}^{2} (x_i(t) \text{ under } d + \Delta d_j - x_i(t) \text{ under } d)^2} \quad (j = 1, \ldots, 1000) \tag{9.20}$$

The maximal value, mean, and variance of all RSME during the simulation period are compared for different design variables in Figure 9.10, where it is clear that the maximal value, mean, and variance obtained by the presented robust design are smaller than all the other designs. Thus, the presented method can obtain the better robust performance.

Example 9.2: The forging press machine in manufacturing industry, as shown in Figure 9.11, uses a hydraulic driven system, including pump, valve, pipe, and cylinder, to drive the up and down movement of the working plate and to offer the required forging force. In this machine, the oil of all hydraulic cylinders is offered by a single hydraulic drive system and, thus, all hydraulic cylinders may be regarded as a single cylinder whose area is equal to the sum of the area of all hydraulic cylinders. The objective of this design is to make the system easier to control and guarantee the robustness of the whole dynamic process under parameter variations and constraint conditions. ■

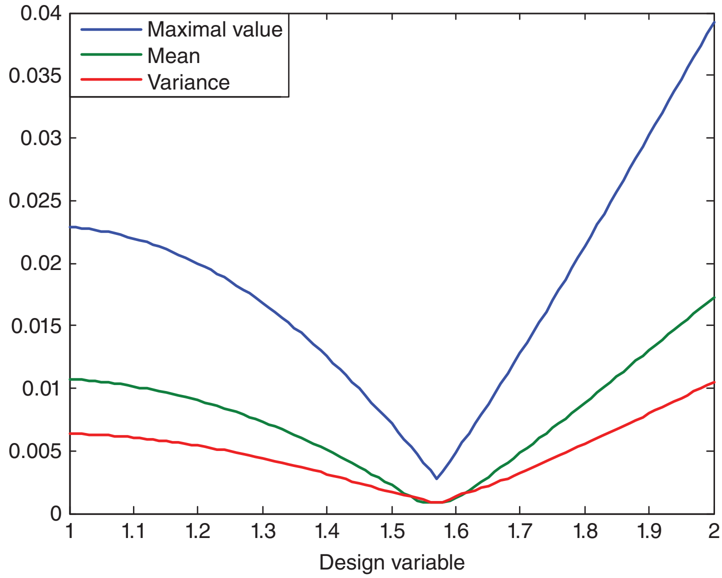
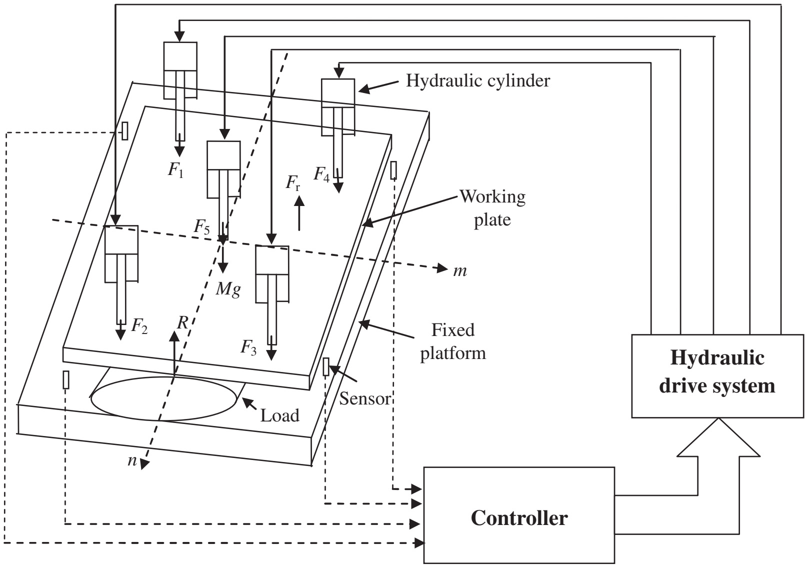


FIGURE 9.10 Maximal value, mean, and variance of RSME under different design variable

(a)



(b)

FIGURE 9.11 The forging press machine system in manufacturing: (a) practical forging press machine; (b) sketch of forging system

According to Newton's second law, the force model of the working plate can be derived as:

$$m\frac{d^2x}{dt^2} = aP - B_c\frac{dx}{dt} + mg - f_c - f_l \tag{9.21}$$

where m is the mass of the working plate, x is the position of the working plate, P is the pressure, B_c is the viscous damping coefficient, and f_c and f_l are the friction force and the known load force, respectively. The parameter a is the area sum of all hydraulic cylinders that is one of design parameters.

The flow model of the hydraulic cylinder can be represented by

$$K_l u = a\frac{dx}{dt} + \frac{V + ax}{\beta_e} \times \frac{dP}{dt} + c_t P \tag{9.22}$$

where V is initial volume, c_t is leak coefficient, and β_e is the spring moment of medium that is another design parameter. The control variable is u with the gain of valve K_l as the control gain.

The friction force can be described as

$$f_c = f_d + (f_s - f_d)e^{-(\frac{v}{v_s})^2} + \sigma v \tag{9.23}$$

where f_d and f_s are the Coulomb and static friction values, and v_s and σ are the Stribeck velocity and the friction coefficient.

The control variable u and the process variable P at the local working point can be expressed as

$$u = u_0 + \Delta u \quad \text{and} \quad P = P_0 + \Delta P \tag{9.24}$$

where u_0 and Δu are the nominal control and the control variation, and P_0 and ΔP are the nominal pressure and the pressure variation, respectively. The nominal values are used to balance the weight, the static friction, and the load force, and the variation values are used to adjust the desirable dynamic trajectory. Thus, we have

$$aP_0 + mg - f_d - f_f = 0 \quad \text{and} \quad K_l u_0 = c_t P_0 \tag{9.25}$$

From Equations 9.21–9.25, the model of the system can be derived as

$$\begin{cases} \dfrac{d^2x}{dt^2} = \dfrac{1}{m}\left(a\Delta P - B_c\dfrac{dx}{dt} - (f_s - f_d)e^{-(\frac{v}{v_s})^2} - \sigma v\right) \\ \dfrac{d\Delta P}{dt} = \dfrac{\beta_e}{V + ax} \times \left(K_l \Delta u - a\dfrac{dx}{dt} - c_t \Delta P\right) \\ v = \dfrac{dx}{dt} \end{cases} \tag{9.26}$$

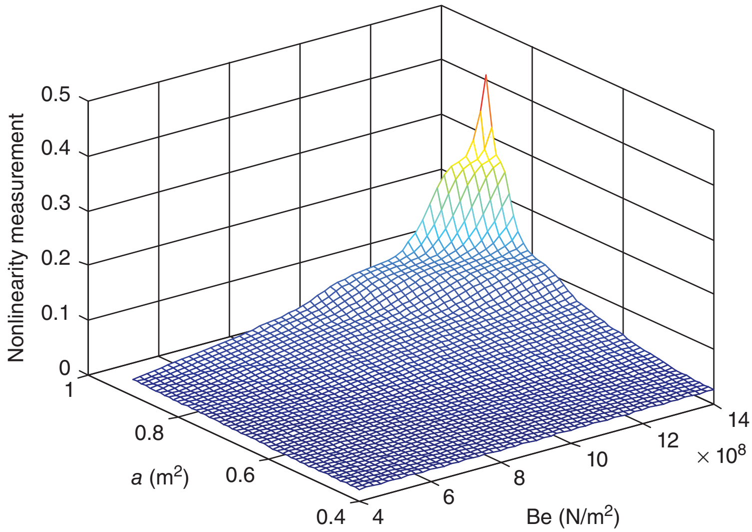


FIGURE 9.12 Nonlinearity measurement

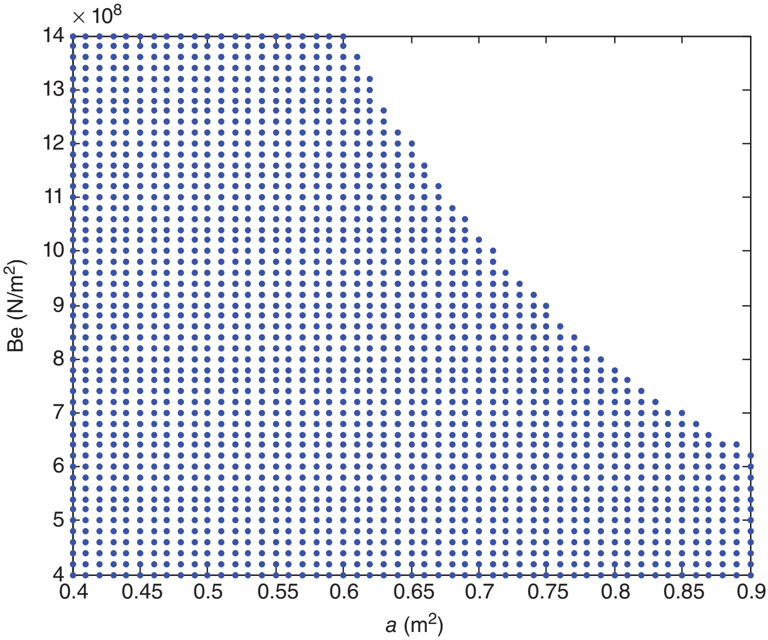


FIGURE 9.13 S_m

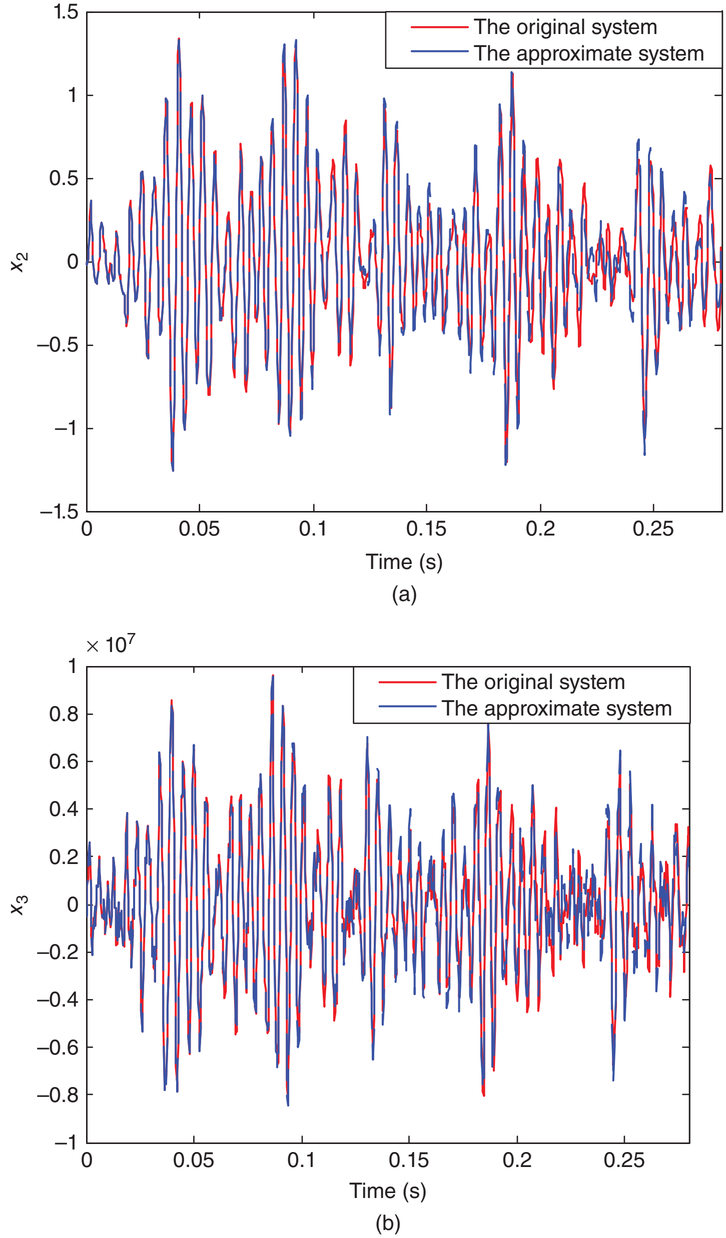


FIGURE 9.14 Approximation performance between the practical system and the linear model: (a) state x_2; (b) state x_3

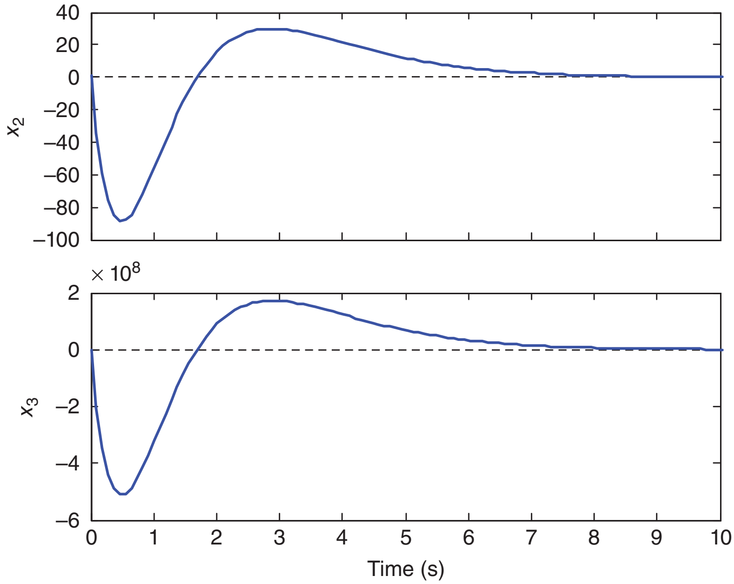


FIGURE 9.15 The nominal performance of the system under the robust design

Define the states $x_1 = x, x_2 = v, x_3 = \Delta P$. The design variables are $d = [a \ \ \beta_e]$, with their nominal values chosen from the design space $a \in [0.4, 0.9]$ and $B_e \in [4 \times 10^8, 1.4 \times 10^9]$. However, there are uncontrollable variations Δd around the nominal values due to manufacturing errors and environment variations. To make the system easy to control, a set of suitable design variables should be found to have a well linear approximation of the system. Then the control gain K could be figured through the integration with robust design for the desired dynamic performance.

9.4.4 Design for Control

The desired value ϕ_0 is set as 0.1. The nonlinearity measurement and the suitable variable space S_m are shown in Figures 9.12 and 9.13, respectively. The blank space stands for infeasible designs and the shadow space stands for feasible designs in Figure 9.13. Moreover, the approximation performance between the practical system and the linear model under $d = [0.4, 4 \times 10^8]$ is shown in Figure 9.14, from which it is clear that the linear model can represent the forging press system well.

9.4.5 Robust Dynamic Design and Verification

Suppose that a good dynamic response requires the desired poles to be on -1, -1.2, and -1.4. In order to achieve this pole placement, the nominal design is first to assign the poles of the nominal system to be on the desired positions. Once the desired poles are given, the nominal parameter space S_n is calculated from Equation 9.9. Then,

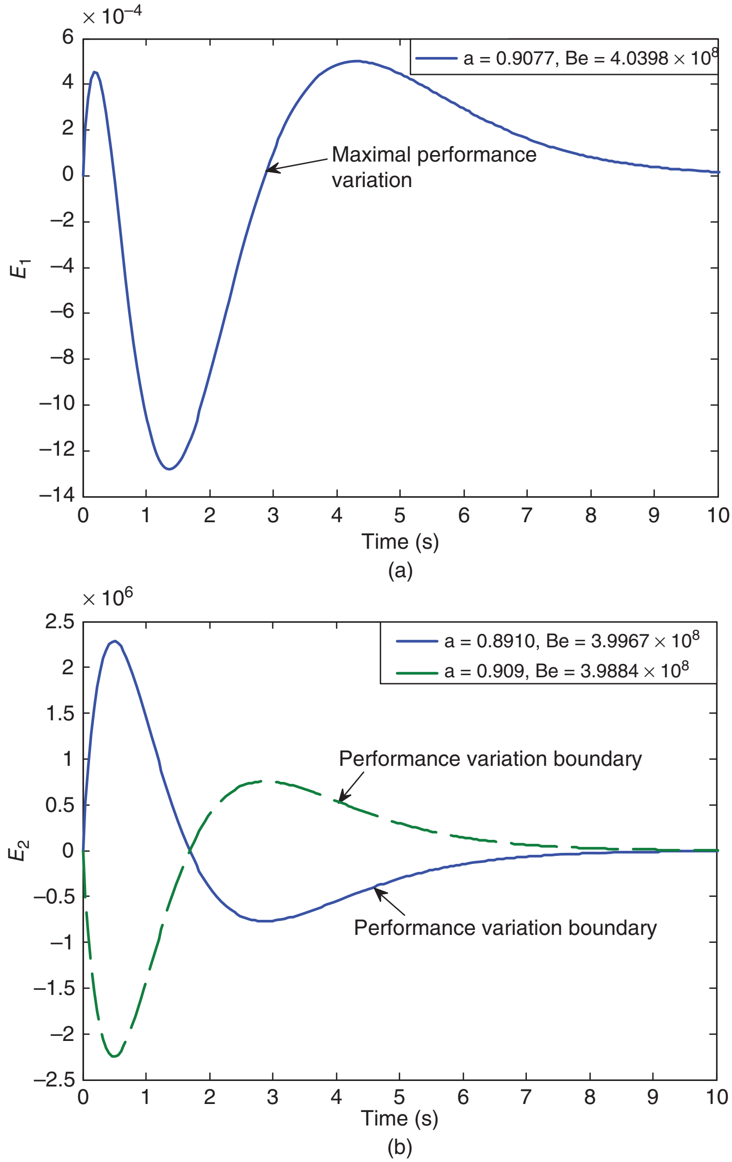


FIGURE 9.16 Performance variation caused by parameter variation Δd; (a) E_1; (b) E_2

the robust dynamic design can be obtained from this nominal parameter space S_n by solving the optimization problem (Equation 9.16). The derived design solutions are $d = [0.9,\ 4 \times 10^8]$ and $K = [0.000007 \quad -0.915 \quad -0.00000034]$, and the poles of the nominal system obtained are -1, -1.2, and -1.399999. The proposed design has achieved the goal. Moreover, the transient response of the nominal system under the robust parameters is shown in Figure 9.15. Obviously, the system is stable and can converge to the equilibrium quickly.

For robustness test, assume that parameter variations Δd randomly vary in $(-0.01d^*, 0.01d^*)$ where d^* is the robust design, then 1000 samples are be used to evaluate the effectiveness of the proposed method. As shown in Figure 9.16, the bounds of performance variations (E_1 and E_2) are smaller than 0.5% of the nominal performance of the system in Figure 9.15. Thus, the proposed approach is very robust to parameter variations.

9.5 SUMMARY

In this chapter, an integrated design and control method is presented for the nonlinear system with hybrid discrete/continuous variables. With the robust pole placement, the design-for-control approach can effectively obtain a well linear approximation for a complex system. Afterwards, the designed system can be effectively controlled by a relatively simple controller. More importantly, this integrated design and control approach can easily achieve the robust pole placement since it has combined the merits of both robust design and control. The simulation results show that this method can obtain the desired performance even by using a simple feedback controller.

CHAPTER 10

INTELLIGENCE-BASED HYBRID INTEGRATION

The method presented in Chapter 9 is suitable for the simple system working in a small operation region. This chapter will develop an integration method for the strongly nonlinear process with unmeasured overall performance to work in a lager operation region. Fuzzy modeling technique is first used to approximate the nonlinear system in a large operation domain, upon which system design and process control can be further integrated for the desired overall performance. A PSO-based hierarchical optimization method is proposed to explore the optimal solution of this complex design and control integration.

10.1 INTRODUCTION

The integrated design and control methods introduced in Chapter 9 can only handle the weak nonlinear system because a linear nominal model of the system is used at the given operating point. This integration method may not be applicable to the system working in a large operating region, which is a common practice in manufacturing. On the other hand, one of the most difficult problems in manufacturing is that the ultimate target or the overall performance of the system is difficult to measure and cannot be directly controlled. The real manufacturing system can be simplified as the hybrid system in Figure 10.1. The system at the machine-level is a nonlinear but continuous process to produce the required product. At the supervision level, the ultimate goal

System Design and Control Integration for Advanced Manufacturing, First Edition. Han-Xiong Li and XinJiang Lu.

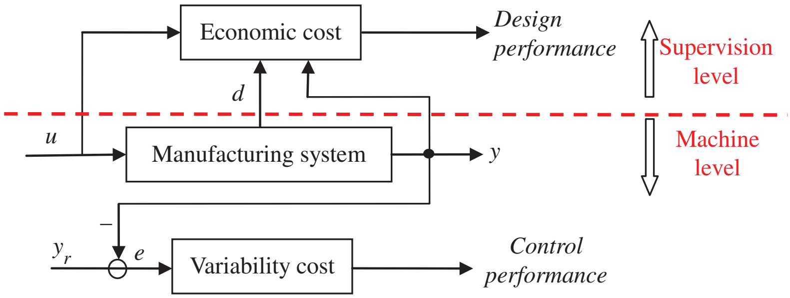


FIGURE 10.1 The hybrid system in manufacturing

of the production is to meet the specific high level criteria, for example, economic criteria in manufacturing. The overall performance of manufacturing actually takes into account both the continuous process output and the static economic index. This kind of system has two level objectives, which will be much more complex to handle than the one-level system discussed in previous chapters. The dynamic performance of this hybrid system depends on not only the low level process control but also the high level system design. So far, there is little research carried out in this aspect.

In the past decades, much progress has been made in controller design of nonlinear systems using the so-called Takagi–Sugeno (T–S) fuzzy model (Takagi and Sugeno, 1985) since it can combine the merits of both fuzzy logic theory and the linear system theory (Takagi and Sugeno, 1985; Tanaka and Wang, 2001; Wu and Li, 2008; Tseng, Chen, and Uang, 2001). It is well known that the fuzzy system provides a simple and straightforward way to decompose the complex task into a group of simple tasks. The T-S fuzzy model has been reported to smoothly approximate any nonlinear system in a large working region (Tanaka and Wang, 2001). However, the application of fuzzy modeling and control for the integrated design and control has not yet been reported for the hybrid system as shown in Figure 10.1.

Generally, optimization problems in the integrated design and control are often too complex to solve using analytical methods because the objective function formulated may be nonconvex or nondifferential (Schluter et al., 2009). Particle swarm optimization (PSO) (Eberhart and Kennedy, 1995) is a good alternative tool for this kind of optimization problems. Since the particle swarm method is inspired by social behavior of a flock of birds and insect swarms, the PSO is not affected by the nonlinearity, nonconvex, and nondifferential property of the problem and can converge to the optimal solution when most of the analytical methods fail (Valle et al., 2008). Therefore, it can be effectively applied to different optimization problems with advantages over many other similar optimization techniques, like the genetic algorithm (GA) (Clerc, 2006; Naka et al., 2003; Valle et al., 2008). So far, this PSO has not yet been applied to system design and control integration for the proposed hybrid system.

In this chapter, intelligence-based design and control integration will be proposed for the manufacturing system in Figure 10.1 that has a hybrid discrete/continuous dynamics. The fuzzy modeling method is first used to approximate the nonlinear process in a large operating region, upon which fuzzy control rules are developed to stabilize the process with the desired robust tracking performance. Then, the steady-state economic design and the controller design are integrated into a unified optimization, which will be solved by a PSO-based hierarchical optimization method. This intelligent solution can achieve the desired economic performance as well as the satisfactory dynamic performance. Finally, the proposed method is compared with the traditional sequential design method and a traditional integration method on controlling the temperature profile of a curing process in IC packaging industry.

10.2 PROBLEM IN HYBRID SYSTEM IN MANUFACTURING

Generally, a nonlinear process with discrete design parameter d and continuous control variable can be described in the following mathematical formula:

$$f(\dot{x}(t), x(t), y(t), u(t), d, w(t)) = 0 \tag{10.1}$$

where $f(\cdot)$ is nonlinear function; $x \in R^n$, $u \in R^{n_u}$, and $y \in R^{n_y}$ represent state vector, control variables, and process outputs, respectively; $d \in R^{n_d}$ denotes discrete design variables which are fixed during the entire period of operation; and w is disturbance. This nonlinear process is supposed to work in a large operating region.

The manufacturing system should be designed from both machine level and supervision level in terms of two different objectives: economic cost and variability cost, as illustrated in Figure 10.1. The overall performance of the hybrid system in manufacturing should incorporate both the variability cost and the economic cost. The variability cost is function of the dynamic performance of the process, while the economic cost covers the process capital and the operating cost.

Since the economic cost is static and closely related to steady-state performance of the process, it can be designed by selecting the optimal discrete parameter d in the system. The design problem for economic cost can be expressed in the following steady-state form:

$$\begin{aligned} &\min_{d} \quad EC(x_{static}, y_{static}, d) \\ &s.t. \quad f(\dot{x}, x, y, u, d, w) = 0 \qquad \text{(Prediction of the steady-state behavior)} \end{aligned} \tag{10.2}$$

where $EC(x_{static}, y_{static}, d)$ is a steady-state economic cost that includes the process capital and the operating cost. The optimization (Equation 10.2) can guarantee an optimal design performance. However, the solution may not prevent the poor process dynamics, such as long settling time or even unstable behavior (Grosch, Monnigmann, and Marquardt, 2008). This is because the dynamic performance of the process is not considered in design optimization.

In control fields, a satisfactory dynamic performance of the process can be easily achieved by optimizing the following variability cost

$$\begin{aligned}&\min_{d,X,Y_1,\ldots,Y_r} \quad VC(e(t))\\&s.t.\ f(\dot{x},x,y,u,d,w)=0\\&\text{Controller design, Stalibity, Robust performance}\end{aligned} \tag{10.3}$$

where the variability cost $VC(e(t))$ is a function of the tracking error $e(t)$ of the process. The optimization (Equation 10.3) can guarantee the stability and robustness of the process under the external disturbance.

From the point of view of production, the system should have not only a desirable dynamic performance but also a small economic cost. Process control engineers can control the dynamic process well but may not have a global view over the manufacturing system. On the other hand, optimal design of economic cost only may cause undesirable dynamics of the process. Obviously, a lack of proper consideration of both criteria may lead to an undesirable production. Thus, an effective integration of design and control method should be developed to consider the low level nonlinear process control as well as the high level system design simultaneously.

10.3 INTELLIGENCE-BASED HYBRID INTEGRATION

10.3.1 Intelligent Process Control

10.3.1.1 Fuzzy Modeling It is well known that fuzzy modeling has a good ability to approximate any nonlinear process in a large operating region. Thus, a fuzzy model will be used to approximate the nonlinear process (Equation 10.1). First, a simple linear model, usually called the local model, is obtained at each operating point. Then the fuzzy technique is used to integrate these local models into a global model, which can approximate any nonlinear process in a large operating region. Moreover, this fuzzy model will be easily controlled by the well-known fuzzy control system because both systems can be designed using the same fuzzy theory.

The nonlinear process model can be represented by the following fuzzy system:

Model rule *i*:
IF $z_1(t)$ is $z_{1,i}$ and … and $z_n(t)$ is $z_{n,i}$
THEN

$$\begin{cases}\dot{x}(t)=A_i x(t)+B_i u(t)+w(t)\\ y(t)=C_i x(t)\end{cases} \quad i=1,2,\ldots,r \tag{10.4}$$

where z_j and $z_{j,i}$ are the premise variable of the process and its fuzzy set respectively, r is the number of IF-THEN rules of the process, A and B are the known matrices obtained from Taylor expansion in the local operating point.

The final fuzzy model is derived as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))[A_i x(t) + B_i u(t) + w(t)] \\ y(t) = \sum_{i=1}^{r} h_i(z(t)) C_i x(t) \end{cases} \tag{10.5}$$

where

$$\mu_i(z(t)) = \prod_{j=1}^{n} M_{j,i}(z_j(t))$$

$$h_i(z(t)) = \frac{\mu_i(z(t))}{\sum_{i=1}^{r} \mu_i(z(t))}$$

The term $M_{j,i}(z_j(t))$ is the grade of membership of $z_j(t)$ in the ith rule. Since

$$\mu_i(z(t)) > 0$$

$$\sum_{i=1}^{r} \mu_i(z(t)) > 0 \;\; i = 1, 2, \ldots, r$$

We have

$$\sum_{i=1}^{r} h_i(z(t)) = 1 \text{ and } 0 < h_i(z(t)) \quad i = 1, 2, \ldots, r$$

The state space model of each rule in the fuzzy model (Equation 10.4) describes the process behavior at a small neighbourhood of the nominal operating point. Since there are many rules for describing different nominal operating points, the fuzzy model (Equation 10.5) can approximate any nonlinear process model in a large working region.

10.3.1.2 Fuzzy Control In practical application, tracking performance is often one of the control objectives. The following model can be considered as the tracking reference:

$$\dot{x}_r(t) = A_r x_r(t) + B_r r(t) \tag{10.6}$$

where $x_r \in R^{n_r}$ is the reference state, and $r(t)$ is the reference input, A_r and B_r are specific matrices with appropriate dimensions, and $x_r(t)$ is a desired trajectory for $x(t)$ to track.

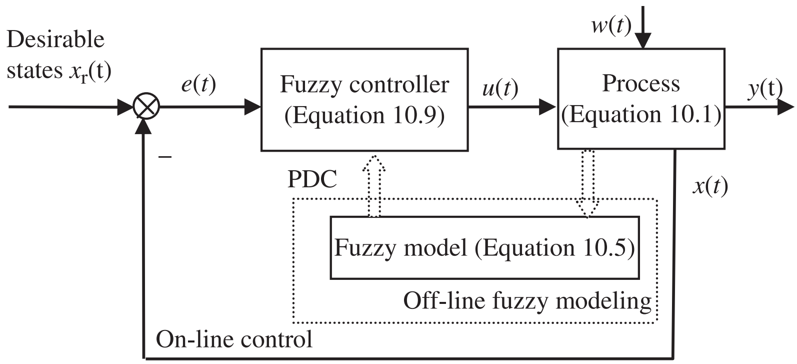


FIGURE 10.2 Fuzzy-model-based fuzzy control

The robust tracking performance under external disturbance may be achieved by using the following H_∞ performance:

$$\int_0^{t_f} [e^T(t)Qe(t)]dt \leq e^T(0)Pe(0) + \gamma^2 \int_0^{t_f} [\tilde{w}(t)^T \tilde{w}(t)]dt \tag{10.7}$$

where $e(t) = x_r(t) - x(t)$ is the tracking error, and $0 \leq Q = C_Q^T C_Q \in R^{n\times n}$ is the weighting matrix that are specified beforehand according to the design purpose, $0 \leq P \in R^{n\times n}$ is an unknown matrix, $\gamma > 0$ is a prescribed attenuation level, and $\tilde{w}(t)$ is equal to $[\,w(t)\; x_r(t)\,]^T$. In general, it is desirable to make γ as small as possible to achieve the optimal disturbance attenuation performance.

Given the fuzzy model (Equation 10.5), fuzzy control is a natural selection to obtain the system stability and robustness because both of them can be designed using the same fuzzy theory. The controller structure is shown in Figure 10.2.

Based on the parallel distributed compensation scheme (PDC) (Tanaka and Wang, 2001), the following fuzzy control law for the fuzzy model (Equation 10.5) is developed as

Model rule *i*:
IF $z_1(t)$ is $z_{1,i}$ and ... and $z_n(t)$ is $z_{n,i}$
THEN

$$u(t) = K_i e(t) \tag{10.8}$$

where $K_i \in R^{n_u\times n}$ is the controller gain.

Obviously, the fuzzy control law may be represented by

$$u(t) = \sum_{i=1}^{r} h_i(z(t))K_i e(t) \tag{10.9}$$

Inserting the control law (Equation 10.9) into Equation 10.5, the closed-loop system may be written as

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))[A_i x(t) + B_i K_j e(t) + w(t)] \\ y(t) = \sum_{i=1}^{r} h_i(z(t))C_i x(t) \end{cases} \tag{10.10}$$

Theorem 10.1: Consider the process model (Equation 10.1) and the fuzzy model (Equation 10.5). Given the fuzzy controller (Equation 10.9), if there exists a common matrix $P > 0$ satisfying

$$\begin{bmatrix} (A - C_v)X + X^T(A - C_v)^T - B(d)Y_w - Y_w^T B^T(d) & \\ \quad + \gamma^{-2}(A - C_v)(A - C_v)^T + 2\gamma^{-2}I & X^T \\ X & -Q^{-1} \end{bmatrix} < 0 \tag{10.11}$$

with $X = P^{-1}, Y_w = K_w X$.

Then the closed-loop model (Equation 10.10) is exponentially stable in the absence of $\tilde{w}(t)$ and the H_∞ control performance (Equation 10.7) is guaranteed in the presence of $\tilde{w}(t)$.

Proof: Choose a Lyapunov function candidate as

$$V(t) = e^T(t)Pe(t) \tag{10.12}$$

Calculate the derivative of $V(t)$ along the trajectory of system (Equation 10.10) and yield

$$\begin{aligned} \dot{V}(t) = \sum_{v=1}^{r}\sum_{w=1}^{r} & h_v(x(t))h_w(x(t))\big[e^T(t)(P\tilde{A} + \tilde{A}^T P)e(t) - x_r^T(t)(A - C_v)^T Pe(t) \\ & - e^T(t)P(A - C_v)x_r(t) - w^T(t)Pe(t) - e^T(t)Pw(t)\big] + \dot{x}_r^T(t)Pe(t) \\ & + e^T(t)P\dot{x}_r(t) \end{aligned} \tag{10.13}$$

$$\text{with } \tilde{A} = A - C_v - B(d)K_w \tag{10.14}$$

Adding and subtracting some terms on the right side of the equality (Equation 10.13), the equality (Equation 10.13) can be rewritten as

$$\begin{aligned}
\dot{V}(t) = & \sum_{v=1}^{r}\sum_{w=1}^{r} h_v(x(t))h_w(x(t))[e^T(t)(P\tilde{A} + \tilde{A}^T P)e(t) - \gamma^2(x_r(t) \\
& + \gamma^{-2}(A - C_v)^T Pe(t))^T(x_r(t) + \gamma^{-2}(A - C_v)^T Pe(t)) + \gamma^2 x_r^T(t)x_r(t) \\
& + \gamma^{-2}e^T(t)P^T(A - C_v)(A - C_v)^T Pe(t) - \gamma^2(w(t) + \gamma^{-2}Pe(t))^T(w(t) \\
& + \gamma^{-2}Pe(t)) + \gamma^2 w^T(t)w(t) + \gamma^{-2}e^T(t)P^T Pe(t)] - \gamma^2(\dot{x}_r(t) - \gamma^{-2}Pe(t))^T(\dot{x}_r(t) \\
& - \gamma^{-2}Pe(t)) + \gamma^2\dot{x}_r^T(t)\dot{x}_r(t) + \gamma^{-2}e^T(t)P^T Pe(t) \\
& \le \sum_{v=1}^{r}\sum_{w=1}^{r} h_v(x(t))h_w(x(t))[e^T(t)\zeta(t)e(t) + \gamma^2\tilde{w}^T(t)\tilde{w}(t)]
\end{aligned} \tag{10.15}$$

where $\zeta(t) = P\tilde{A} + \tilde{A}^T P + \gamma^{-2}P^T(A - C_v)(A - C_v)^T P + 2\gamma^{-2}P^T P$

Obviously, if the following inequality holds:

$$\zeta(t) < -Q \tag{10.16}$$

then, from the inequality (Equation 10.15), we get

$$\dot{V}(t) \le -e^T(t)Qe(t) + \gamma^2\tilde{w}^T(t)\tilde{w}(t) \tag{10.17}$$

when $\tilde{w}(t) = 0$, the inequality (Equation 10.17) may be written as

$$\dot{V}(t) \le -e^T(t)Qe(t) \le 0 \tag{10.18}$$

Thus, if the inequality (Equation 10.16) holds and $\tilde{w}(t) = 0$, then $\dot{V}(t) \le 0$ and the system is stable.

Assume that the inequality (Equation 10.16) holds and $P > 0$. From the inequality (Equation 10.16) and the equality (Equation 10.14), we have

$$\begin{aligned}
& P(A - C_v - B(d)K_w) + (A - C_v - B(d)K_w)^T P + \gamma^{-2}P^T(A - C_v)(A - C_v)^T P \\
& + 2\gamma^{-2}P^T P + Q < 0
\end{aligned} \tag{10.19}$$

Pre- and post-multiplying X^T and X with $P = X^{-1}$

$$\begin{aligned}
& (A - C_v - B(d)K_w)X + X^T(A - C_v - B(d)K_w)^T + \gamma^{-2}(A - C_v)(A - C_v)^T \\
& + 2\gamma^{-2}I + X^T QX < 0
\end{aligned} \tag{10.20}$$

Let $Y_w = K_w X$. Then the inequality (Equation 10.20) can be rewritten as

$$(A - C_v)X + X^T(A - C_v)^T - B(d)Y_w - Y_w^T B^T(d) + \gamma^{-2}(A - C_v)(A - C_v)^T + 2\gamma^{-2}I + X^T QX < 0 \tag{10.21}$$

The inequality (Equation 10.21) may be transformed into LMI form

$$\begin{bmatrix} (A - C_v)X + X^T(A - C_v)^T - B(d)Y_w - Y_w^T B^T(d) + \gamma^{-2}(A - C_v)(A - C_v)^T + 2\gamma^{-2}I & X^T \\ X & -Q^{-1} \end{bmatrix} < 0 \tag{10.22}$$

Thus, the stable condition (Equation 10.16) is transformed into Equation 10.12 (the same with Equation 10.22), which can be solved by LMI.

If the inequality (Equation 10.12) is satisfied, the inequality (Equation 10.17) will hold. Integrating Equation 10.17 from $t = 0$ to $t = t_f$ yields

$$V(t_f) - V(t_0) \leq -\int_{t_0}^{t_f} e^T(t)\, Qe(t)\, dt + \gamma^2 \int_{t_0}^{t_f} \tilde{w}^T(t)\, \tilde{w}(t)\, dt \tag{10.23}$$

Since the Lyapunov function $V(t_f) > 0$, the inequality (Equation 10.23) is rewritten as

$$\int_{t_0}^{t_f} e^T(t)\, Qe(t)\, dt \leq V(t_0) + \gamma^2 \int_{t_0}^{t_f} \tilde{w}^T(t)\, \tilde{w}(t)\, dt \tag{10.24}$$

Thus, we can get the H_∞ tracking performance (Equation 10.7) from Equation 10.24.

In addition, when $\tilde{w}(t) = 0$, the inequality (Equation 10.17) may be written as

$$\dot{V}(t) \leq -e^T(t)Qe(t) \leq -\lambda_{\min}(Q)e^T(t)e(t) \leq -\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}V(t) \tag{10.25}$$

From the above inequality (Equation 10.25), we have $V(t) \leq V(0)e^{-\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}t}$, so that $\|e(t)\| \leq \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}e^{-\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}t}\ \|e(0)\|$ for all trajectories.

Therefore, the closed-loop fuzzy system (Equation 10.10) with $\tilde{w}(t) = 0$ is exponentially stable. ■

The upper bound of the robust tracking performance (Equation 10.7) can be minimized by solving the following constraint optimization:

$$\begin{aligned} &\min_{X,Y_1,\ldots,Y_r} \gamma^2 \\ &\textit{subject to} \quad \gamma > 0, X > 0, \text{ the inequality (10.11)} \end{aligned} \tag{10.26}$$

This optimization problem can be solved by linear matrix inequality (LMI), and its solution will guarantee that the system is stable and has a desirable robust tracking performance. After obtaining $X, Y_1, \ldots, Y_r$ from Equation 10.26, the controller gain is expressed as

$$K_j = Y_j X^{-1} \tag{10.27}$$

10.3.2 Hybrid Integration Design

In order to have optimal overall performance, both design and control constraints are needed to consider simultaneously. Thus, the steady-state economic cost (Equation 10.2) and the variability cost (Equation 10.3) are integrated into a unified objective function as follows:

$$\begin{aligned} &\min_{d,X,Y_1,\ldots Y_r} J = EC(x_{static}, y_{static}, d) + VC(e(t)) \\ &s.t.\ f(\dot{x}, x, y, u, d, w) = 0 \\ &\text{the equalities (10.9) and (10.27)} \\ &\text{the inequality (10.11)} \\ &\gamma < \gamma_0 \end{aligned} \tag{10.28}$$

where the equalities (Equations 10.9 and 10.27) are the control rules and the control gains, respectively, and the inequality (Equation 10.11) is the requirement of the stability. The constraint $\gamma < \gamma_0$ can guarantee the robust tracking performance, with γ_0 as a limiting value determined by users for the acceptable attenuation of disturbance.

The proposed integration of design and control contains two main parts, as shown in Figure 10.3.

- Part 1 is to design the steady-state economic cost. It minimizes the process capital and the operating cost (Equation 10.2) by optimizing the design variables d of the steady-state process. The optimal design performance can be obtained.
- Part 2 is to design the control system, which includes the process modeling and controller design. The nonlinear process is globally approximated by the fuzzy modeling method. Then, based on this fuzzy model, a fuzzy controller is derived to stabilize the system and minimize the variability cost, and guarantee the feasibility as well.

The optimal solution of this hybrid integration (Equation 10.28) can simultaneously achieve the optimal performance for both high level system design and the low level process control.

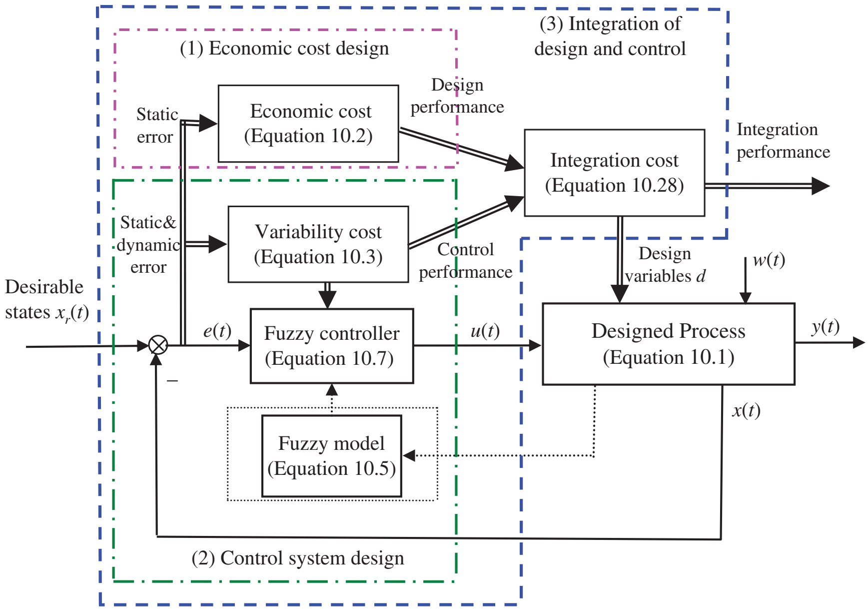


FIGURE 10.3 Integration framework for the hybrid system

10.3.3 Hierarchical Optimization of Integration

Obviously, integration (Equation 10.28) is a nonlinear optimization problem constrained by both design and control requirements. The identification of the controller gains and the design variables using mathematical programming algorithms is very difficult due to discontinuous and nonconvex property in the integration space (Malcolm et al., 2007). Thus, it is nearly impossible to optimize design variables and fuzzy controller gains simultaneously using traditional optimization methods.

In order to make the integration problem easier to tackle, a hierarchical optimization framework proposed by Malcolm et al. (2007) is employed to decompose the integration problem into two nested optimizations: the embedded control optimization (inner loop) and the master design optimization (outer loop), as shown in Figure 10.4.

10.3.3.1 Embedded Control Optimization Given the design variables, the embedded control optimization (inner loop) is to guarantee system stability and robustness. This optimization strategy is shown in Figure 10.5, and summarized as follows:

Step 1 (Initialization): Design variables are determined by master design optimization and the weighting matrix Q is specified as performance requirement.

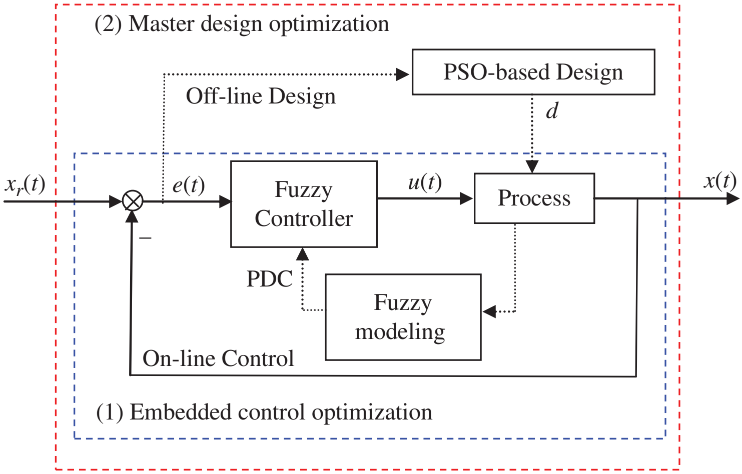


FIGURE 10.4 Intelligence-based hierarchical optimization

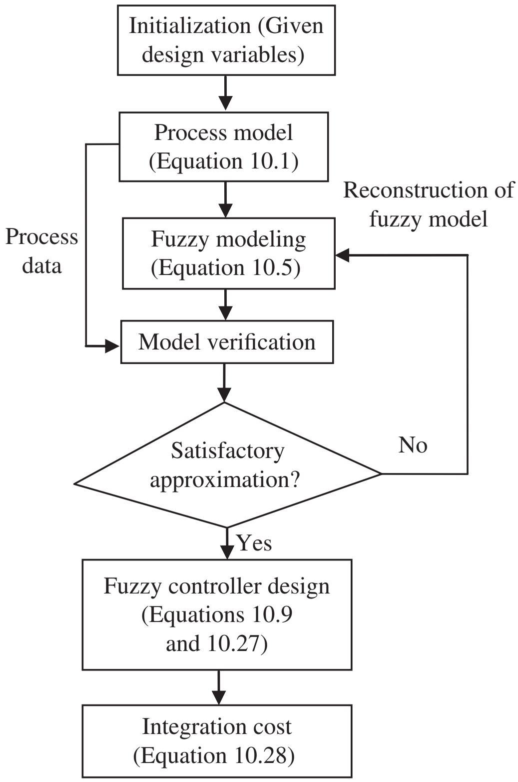


FIGURE 10.5 Embedded control optimization

Step 2 (Fuzzy modeling): The nonlinear process model can be obtained according to process knowledge. Then it will be modeled by fuzzy modeling method, and represented as the fuzzy model (Equation 10.5).

Step 3 (Model verification): Input/output data of the nonlinear process are used to verify the fuzzy model. If the approximation error is large, then return to Step 2 to reconstruct the fuzzy model by adding more rules r and adjusting local models.

Step 4 (Fuzzy control design): Based on the obtained fuzzy model, the fuzzy controller is determined by solving Equations 10.9 and 10.27 using LMI. Then, the integration cost can be obtained from the feedback loop system.

10.3.3.2 Master Design Optimization The master design optimization is conducted in outer loop, as shown in Figure 10.6. It aims to determine the optimal design variables by solving the integration problem (Equation 10.28), where the control variable u and the attenuation level γ have already been obtained in the embedded control optimization. Usually, this kind of optimization is too complex for analytical methods to handle since the problem could be nonconvex or nondifferential. Here, the intelligent PSO method presented in Chapter 7 is used to solve this master optimization.

In summary, since the control system design is embedded into the master design optimization, the successive iterations of the master design loop will gradually improve the integration performance. Since the PSO-based design is integrated with the fuzzy modeling/control, this integration method combines the merits of both fuzzy modeling/control and PSO. Thus, it is able to deal with the complex nonlinear problem in a large operating region, and can achieve the desired design performance for the high level system as well as the satisfactory control performance of the low level process.

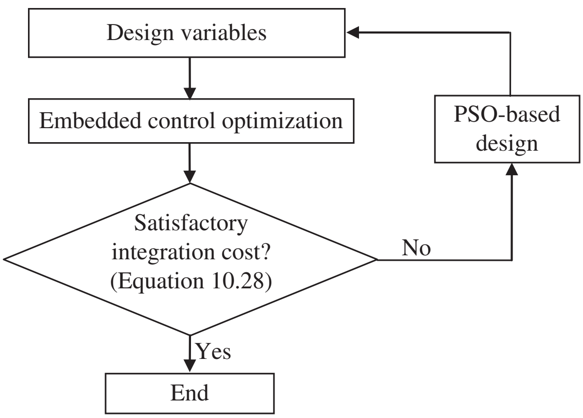


FIGURE 10.6 Master design optimization

10.4 CASE STUDY

Consider the curing oven design given in Example 1.4 in Section 1.1.3 of Chapter 1. The objective is to optimize design variables $d = [\theta, H]$ and control the power of the heaters to obtain a satisfactory tracking performance and a uniform temperature distribution on the lead frame (LF).

The view factor $F_{i,j}(d)$ is expressed as

$$F_{i,j}(d) = F(x_i, y_j, 0) + F(x_i, b - y_j, 0) \tag{10.29}$$

where b and $(x_i, y_j, 0)$ are the breadth of heater block and the coordinate of the zone (i, j) as shown in Figure 1.5, respectively, and the function $F(x, y, 0)$ can be derived according to the definition of view factor and Stokes's theorem (Siegel and Howell, 2002) and expressed as

$$F(x, y, 0) = F_1 + F_2 + F_3 + F_4$$

with

$$F_1 = \frac{\frac{b}{2} - y}{2\pi\sqrt{\left(\frac{b}{2} - y\right)^2 + \left(H + \frac{b}{2}ctg\left(\frac{\theta}{2}\right)\right)^2}} \left(\text{atan} \frac{l - x}{\sqrt{\left(\frac{b}{2} - y\right)^2 + \left(H + \frac{b}{2}ctg\left(\frac{\theta}{2}\right)\right)^2}} \right.$$
$$\left. + \left(\text{atan} \frac{x}{\sqrt{\left(\frac{b}{2} - y\right)^2 + \left(H + \frac{b}{2}ctg\left(\frac{\theta}{2}\right)\right)^2}} \right) \right)$$

$$F_2 = \frac{y}{2\pi\sqrt{y^2 + H^2}} \left(\text{atan} \frac{l - x}{\sqrt{y^2 + H^2}} + \text{atan} \frac{x}{\sqrt{y^2 + H^2}} \right)$$

$$F_3 = -\frac{l - x}{2\pi\sqrt{\eta^2 + (l - x)^2 + (l - x)^2(tg\theta)^2}} \times \left(\text{atan}\left(\frac{(-y)(1+(tg\theta)^2)+\eta \times tg\theta}{\sqrt{\eta^2 + (l - x)^2 + (l - x)^2(tg\theta)^2}} \right) \right.$$
$$\left. -\text{atan}\left(\frac{\left(\frac{b}{2} - y\right)\left(1 + (tg\theta)^2\right) + \eta \times tg\theta}{\sqrt{\eta^2 + (l - x)^2 + (l - x)^2(tg\theta)^2}} \right) \right)$$

$$F_4 = \frac{x}{2\pi\sqrt{\eta^2 + x^2 + x^2(tg\theta)^2}} \times \left(\text{atan}\left(\frac{\left(\frac{b}{2} - y\right)\left(1 + (tg\theta)^2\right) + \eta \times tg\theta}{\sqrt{\eta^2 + x^2 + x^2(tg\theta)^2}} \right) \right.$$
$$\left. - \text{atan}\left(\frac{(-y)\left(1 + (tg\theta)^2\right) + \eta \times tg\theta}{\sqrt{\eta^2 + x^2 + x^2(tg\theta)^2}} \right) \right)$$

$$\eta = -yctg\frac{\theta}{2} - H$$

Define

$$x(t) = \left[T_{1,1}(t) \quad \cdots \quad T_{i,p-1}(t) \quad T_{i,p}(t) \quad T_{i+1,1}(t) \quad T_{i+1,2}(t) \quad \cdots \quad T_{n,p}(t)\right]^T,$$
$$w(t) = \left[\hat{w}_{1,1}(t) \quad \cdots \quad \hat{w}_{i,p-1}(t) \quad \hat{w}_{i,p}(t) \quad \hat{w}_{i+1,1}(t) \quad \hat{w}_{i+1,2}(t) \quad \cdots \quad \hat{w}_{n,p}(t)\right]^T,$$

$$z(t) = diag\left(\left[T^3_{1,1}(t) \ \cdots \ T^3_{i,p-1}(t) \ T^3_{i,p}(t) \ T^3_{i+1,1}(t) \ T^3_{i+1,2}(t) \ \cdots \ T^3_{n,p}(t)\right]^T\right) \tag{10.30}$$

where $T_{ij}(t)$ is the temperature at the (i, j) zone on the LF at time t. The system model (Equation 1.5) is rewritten as the vector form

$$\dot{x}(t) = Ax(t) - Cz(t)x(t) + B(d)u(t) + w(t) \tag{10.31}$$

where u is control variable to offer heating power, and matrixes A, $B(d)$, and C can be derived from Equations 1.9 and 10.30.

Obviously, the model (Equation 10.31) has strong nonlinearity and must work in a large operating region (Temperature range: 40°C∼200°C) as it has to track the required temperature profile.

10.4.1 Objective

10.4.1.1 Ideal Process Output The length, breadth, and height of the LF are 240, 90, and 0.2 mm, respectively. The LF is uniformly divided into 36 zones. The desired temperature profile (°C) is given as

$$T_r(t) = \begin{cases} 40 + 7t & \text{for } \ 0 \le t \le 20 \\ 180 & \text{for } \ 21 \le t \le 50 \end{cases} \tag{10.32}$$

10.4.1.2 Design for Economic Cost One important criterion for a curing process to satisfy the high quality packaging is the uniform temperature distribution under the steady state. A desired uniformity is that the static temperature at each point on the LF is close to the desired reference temperature. The most natural goal to obtain this optimal uniformity is to minimize the error between the static temperature of the LF and the desired reference. Thus, the typical steady-state economic cost can be considered as:

$$EC = \delta_1 \left(\frac{1}{30} \int_{t_s=21}^{t=50} \left(\sum_{i=1,n=1}^{6} \sum_{j=1,m=1}^{6} T_{ij}(t_s) - T_{nm}(t_s) \right) dt \right)^{1/2} \tag{10.33}$$

where δ_1 is a conversion factor to convert the temperature distribution performance $(\frac{1}{30}\int_{t_s=21}^{t=50}(\sum_{i=1,n=1}^{6}\sum_{j=1,m=1}^{6}T_{ij}(t_s)-T_{nm}(t_s))dt)^{1/2}$ to the economic cost, and t_s is time when the system settles down in steady state.

10.4.1.3 Design for Process Control Another important criterion is to guarantee that the dynamic error between the maximal temperature $T_{\max}(t)$ and the minimal temperature $T_{\min}(t)$ on the whole LF should be less than 10°C, and the steady-state error between the maximal static temperature $T_{\max,static}$ and the minimal static temperature $T_{\min,static}$ on the whole LF should be less than 5°C. Thus, the constraints considered will be,

$$T_{\max}(t) - T_{\min}(t) \leq 10 \tag{10.34a}$$

$$T_{\max,static} - T_{\min,static} \leq 5 \tag{10.34b}$$

Furthermore, the disturbance is

$$\hat{w}_{i,j}(t) = norm(0, 1) \tag{10.35}$$

where *norm* (0,1) denotes Gaussian distributions with zero mean and the identical standard deviation.

In order to track the required temperature profile, the variability cost $VC(e(t))$ is considered as

$$VC(e(t)) = \delta_2\left(\frac{1}{50}\int_{t=0}^{t=50}\left(\sum_{i=1}^{6}\sum_{j=1}^{6}e_{ij}(t)^T e_{ij}(t)\right)dt\right)^{1/2} \tag{10.36}$$

with spatio-temporal error: $e_{i,j}(t) = T_r(t) - T_{i,j}(t)$.
where δ_2 is a conversion factor to convert the tracking error $(\frac{1}{50}\int_{t=0}^{t=50}(\sum_{i=1}^{6}\sum_{j=1}^{6}e_{ij}(t)^T e_{ij}(t))dt)^{1/2}$ to the variability cost. The conversion factors should be determined according to the economic situation on the manufacturing site. For our example here, the factors δ_1 and δ_2 are set as 1($/°C) and 1.5($/°C), respectively. The objective is to obtain a uniform temperature distribution (Equation 10.33) on the LF and a satisfactory tracking performance (Equation 10.36) through the simultaneous optimization of the controller and the design parameters (θ and H), under the constraints $\theta \in (160°, 180°)$ and $H \in (4\text{ mm}, 14\text{ mm})$.

10.4.2 Integration Method for the Curing Process

Since the curing oven is fully symmetric about x axis and y axis, only a quarter portion of the LF is required for design and control. The nonlinear curing model with uncertainty is first approximated by the fuzzy model (Equation 10.5). The membership function $h_v(x(t))$ for the fuzzy sets of $x(t)$ is shown in Figure 10.7.

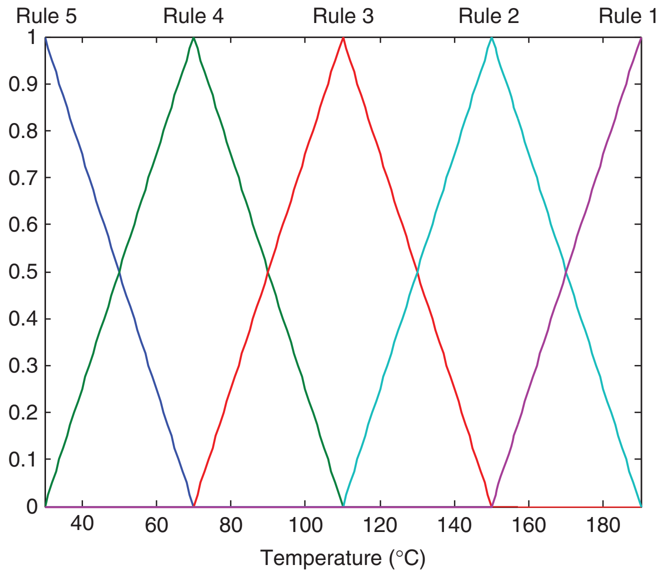


FIGURE 10.7 Membership functions

The fuzzy T–S controller (Equation 10.9) is used to control the fuzzy model (Equation 10.5). Then, 20 particles are employed for PSO in the master design optimization. The velocity and position of the particle v_i are determined by Equations 7.26 and 7.27 with $\delta = 0.7$, $\varphi_1 = \varphi_2 = 1.47$. As shown in Figure 10.8, the iteration process finally converges when the integration cost remains steady. The final optimal

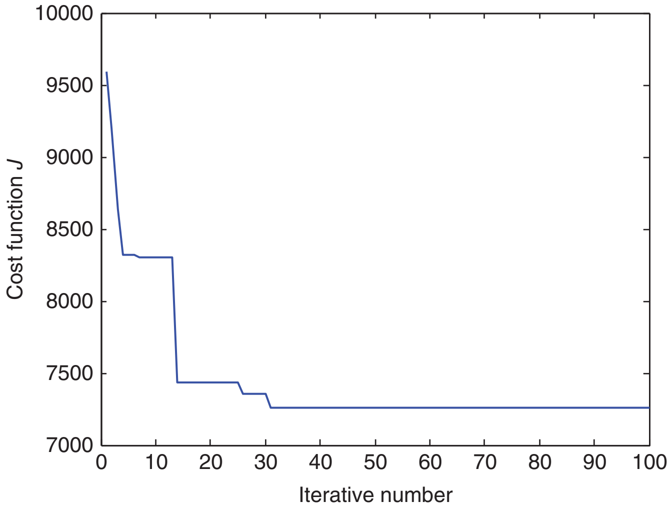


FIGURE 10.8 Iterative process of PSO

design variables $d_I^* = [\theta^*, H^*]$ are equal to [172°, 4.5 mm] under the weighting matrix $Q = 0.1I$ with I as the unit matrix. The controller gains under the optimal design parameters d^* are listed as follows:

$K1$ = [20.4888 20.5920 20.5409 21.1160 21.1725 21.2168 21.1909 21.2750 21.1530]
$K2$ = [20.3419 20.4449 20.3938 20.9658 21.0219 21.0665 21.0403 21.1242 21.0012]
$K3$ = [20.4552 20.5584 20.5072 21.0817 21.1380 21.1824 21.1564 21.2405 21.1182]
$K4$ = [20.6381 20.7417 20.6905 21.2688 21.3256 21.3697 21.3441 21.4283 21.3074]
$K5$ = [20.1817 20.2844 20.2333 20.8019 20.8577 20.9024 20.8760 20.9597 20.8355]

10.4.3 Verification and Comparison

Since the cure temperature can change from 40°C to 200°C, the system has a large working region. The robust tracking performance and the uniform temperature performance are shown in Figure 10.9. In Figure 10.9a, the maximal and minimal temperature trajectory on the whole surface of the LF are very close to the reference signal. In Figure 10.9b, all dynamic temperature differences $e_d(t)$ (defined as $e_d(t) = T_{\max}(t) - T_{\min}(t)$) between the maximal temperature $T_{\max}(t)$ and the minimal temperature $T_{\min}(t)$ on the whole LF are less than 4°C. This means that it has a good variability cost. In Figure 10.9(c), since all steady-state temperature errors $e_{i,j}$ (defined as $e_{i,j}(t_f) = T_r(t_f) - T_{i,j}(t_f)$ with the end time t_f) are less than 3°C, it has a good economic cost. Thus, the design satisfies temperature uniformity (Equation 10.33) under constraints (Equation 10.34). Thus, it is clear from Figure 10.9 that this integration method can obtain the desired robust tracking performance and the uniform temperature distribution as well.

For a better verification, this integration method is compared with the two existing methods. Some performance indexes are set up for an easy comparison as follows:

- Mean squared error

$$MSE = \frac{1}{36 \times 50} \sum_{t=1}^{50} \sum_{i=1}^{6} \sum_{j=1}^{6} e_{i,j}(t)^2$$

- Spatial normalized absolute error

$$SNAE(t) = \frac{1}{36} \sum_{i=1}^{6} \sum_{j=1}^{6} \left| e_{i,j}(t) \right|$$

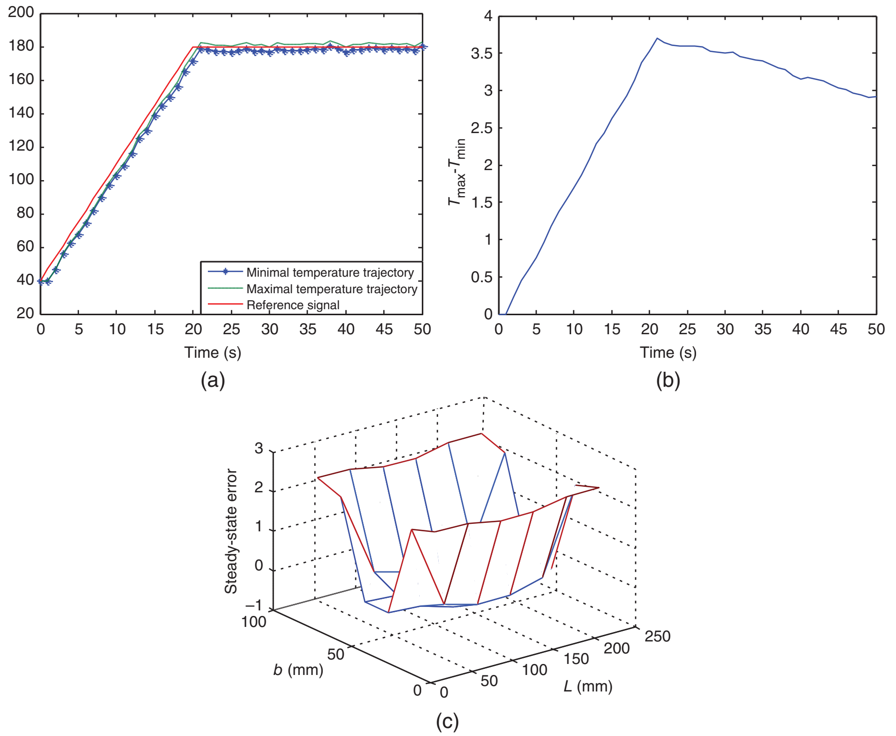


FIGURE 10.9 Robust tracking performance: (a) tracking performance; (b) dynamic temperature difference; (c) steady-state temperature error on the LF

- Time normalized absolute error

$$TNAE(i,j) = \frac{1}{50}\sum_{t=1}^{50} |e_{i,j}(t)|$$

- Difference of time normalized absolute error

$$DTNAE(i,j) = TNAE_T(i,j) - TNAE_P(i,j)$$

where $TNAE_T$ and $TNAE_p$ are the time normalized absolute error obtained by the existing method and the newly proposed integration respectively.

1. Comparison with the traditional sequential design method

The sequential design method first obtains the design variable d through optimizing the steady-state economic design problem (Equation 10.2), followed by the control design for the optimal variability cost (Equation 10.3). The optimal design variable calculated by the sequential method is $d_T^* = [166°, 4\text{ mm}]$.

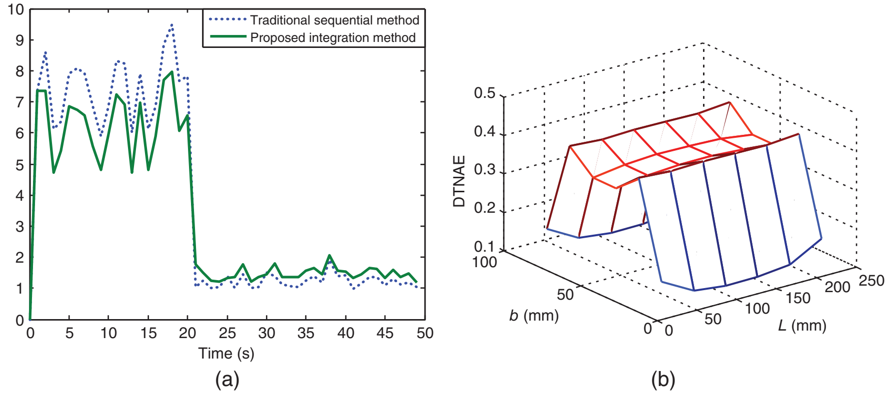


FIGURE 10.10 Performance comparison: (a) *SNAE*; (b) *DTNAE*

The *SNAE* and *DTNAE* are shown in Figure 10.10.

- From Figure 10.10a, we see that the *SNAE* of the proposed integration method is much smaller than the traditional sequential method when the process works in the dynamic environment (from 0s to 20s). This comparison shows that this integration method can improve the dynamic performance and keep a better steady-state performance than the traditional sequential method.
- From Figure 10.10b, since all *DTNAE* are larger than zero, the proposed integration method has a smaller *TNAE* than the traditional sequential method. This comparison shows that the integration method can obtain a better performance than the traditional sequential method.

Finally, *MSE* are compared for both methods, with 4.2855 for the proposed integration method and 4.9117 for the sequential method, respectively. It is clear that the proposed integration method has a better *MSE* than the sequential method.

Based on the above comparisons, it is clear that the proposed integration method has a better performance than the sequential method.

2. Comparison with a traditional integration method

The proposed integration method is compared with the traditional integration method (Meeuse and Tousain, 2002). In Meeuse's integration method, a linear model, which is obtained by linearization-based modeling around the operating point, is first used to approximate the process model. Then the Linear Quadratic Gaussian (LQG) controller and the design variables are designed together based on the obtained linear model. The optimal design variables calculated by Meeuse's integration method are $d^*_{LQG} = [170°, 5\text{ mm}]$.

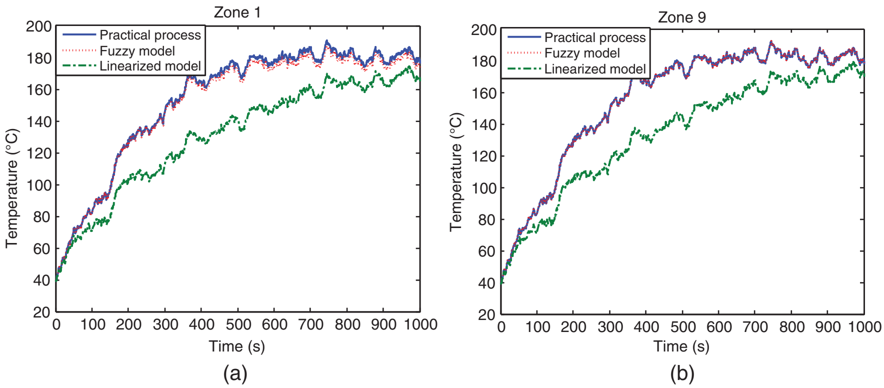


FIGURE 10.11 Performance comparison of practical process and models: (a) model performance in zone 1; (b) model performance in zone 10

- ***Comparison for modeling performance***

Let the control input u randomly vary in [60, 70] and the design variables d are set as [170°, 5 mm]. A total of 1000 samples are taken to compare approximate performance. From Figure 10.11, it is clear that the fuzzy model approximates the nonlinear process better than the linearization-based model around the working point (Meeuse and Tousain, 2002). This is because the fuzzy modeling approach can approximate the nonlinear process well in a large working domain but the linearization-based model does not.

The spatio-temporal error $e_{i,j}(t)$ is defined as the difference between the practical process and the approximate model, and $TNAE_T$ and $TNAE_P$ are the time normalized absolute error obtained by the linearization-based modeling and the fuzzy modeling, respectively. The *SNAE* and *DTNAE* are shown in Figure 10.12. From Figure 10.12a, it is clear that the *SNAE* of the fuzzy modeling method is much smaller than that

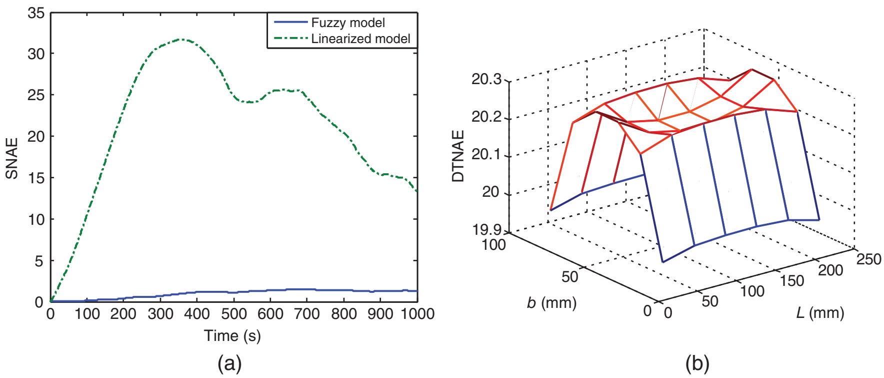


FIGURE 10.12 Comparison of modeling performance: (a) *SNAE*; (b) *DTNAE*

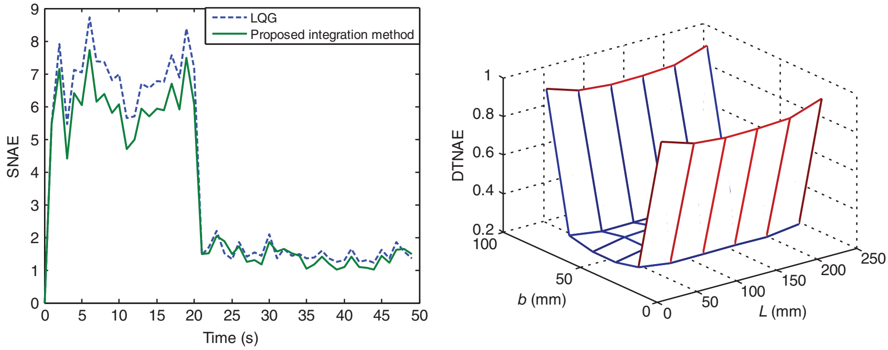


FIGURE 10.13 Comparison of control performance: (a) *SNAE*; (b) *DTNAE*

of the linearization-based modeling in the whole operation. From 10.12b, since all *DTNAE* are larger than zero, the fuzzy modeling method has a smaller *TNAE* than the linearization-based method. This comparison shows that the fuzzy modeling method can obtain better approximation performance than the linearization-based modeling method.

Finally, *MSE* are also compared for both methods, with 0.8923 for the fuzzy modeling method and 30.7271 for the linearization-based modeling method, respectively. It is clear that the fuzzy modeling method has a better *MSE* than the linearization-based modeling method.

- ***Comparison for integration performance***

Let the spatio-temporal error $e_{i,j}(t) = T_r(t) - T_{i,j}(t)$, and $TNAE_T$ and $TNAE_P$ are the time normalized absolute error obtained by Meeuse's integration method and the proposed integration method, respectively. The *SNAE* and *DTNAE* are shown in Figure 10.13. From Figure 10.13a, it shows that the *SNAE* of the proposed integration method is much smaller than that of Meeuse's integration method in the whole operation. In Figure 10.13b, since all *DTNAE* are larger than zero, the proposed method has a smaller *TNAE* than Meeuse's integration method. These comparisons show that the proposed intelligent integration can obtain a better performance than Meeuse's integration method.

Moreover, *MSE* are compared for both methods, with 4.2855 for the proposed integration method and 4.899 for Meeuse's integration method, respectively. It is clear that the intelligent integration method proposed has a better *MSE* than Meeuse's integration method.

Thus, from the above comparisons, it is clear that this new intelligent integration method has a better performance than Meeuse's integration method. This is because the intelligent integration proposed can approximate and control the nonlinear process well, but it is difficult for Meeuse's integration method to approximate and control the nonlinear process well when working in a large operating region.

10.5 SUMMARY

The intelligence-based design and control integration is presented for the hybrid manufacturing system. At the machine level, the fuzzy modeling method can approximate the nonlinear process very well in a large operating region, and fuzzy control can maintain the stability and robustness of the nonlinear process. At the supervision level, the economic cost and other factors will be considered for the overall performance of manufacturing. The hierarchical strategy proposed will optimally integrate high level manufacturing design and low level process control for a better overall performance of the system. The simulation on a snap-curing process in IC packaging industry shows that the proposed method can achieve better overall performance than the traditional approach.

CHAPTER 11

CONCLUSIONS

This chapter summarizes all methods introduced in the book, and discusses future challenges in this area.

11.1 SUMMARY AND CONCLUSIONS

As higher speed, higher precision, and higher intelligence become common requirements in advanced manufacturing, it has led to higher quality design of every component and part in the manufacturing system. One of the serious problems in design is inconsistent performance caused by uncontrollable variations. Robust design and its integration with control are the most important methods commonly used to achieve the robust performance for the system. Studies of robust design and its integration with control become more and more active and important.

After an overview of robust design and its integration with control, the existing robust designs and integrated design and control methods have some limitations, for example, robust design under model uncertainty, robust eigenvalue design, and integration problem for the hybrid system working in a large operating region. The book focuses on developing new robust design methods and new approaches to integrate design and control solutions in order to break through these limitations. The contributions of this book can be briefly summarized as follows:

- First, several novel robust design approaches are proposed to minimize the influence of parameter variations and model uncertainty to the static system.

System Design and Control Integration for Advanced Manufacturing, First Edition. Han-Xiong Li and XinJiang Lu.

- Second, several novel robust design approaches are proposed for the dynamic system to consider system stability and robustness.
- Finally, two novel integration methods are proposed to design and control the hybrid system.

Novel contributions of these new methods can be summarized below.

1. Two novel variable sensitivity robust design approaches are developed to design the robustness of the nonlinear system under small parameter variations. Since the nonlinear system is formulated under a linear structure, it is easy to handle using the existing robust design methods. By minimizing its variable sensitivity matrix, two different robust designs are developed, one in deterministic nature and another in probabilistic nature. These two methods could effectively achieve the robustness for the nonlinear system under deterministic or probabilistic uncertainties despite uncontrollable variations.
2. A new design method is developed for a nonlinear system to maintain robustness under large parameter variations. Since the parameter space is divided into many small subdomains, a local linearization model can be well developed at each subregion. Thus, the nonlinear system can be well described by multiple local models in a large parameter space. With the help of multi-domain modeling, the proposed robust design method could ensure the robustness of the nonlinear system under large parameter variations.
3. Two novel robust design approaches are proposed to design the partially unknown system with variations of design variables. Data-based uncertainty compensation method could extract useful information hidden in the data to compensate the effect of model uncertainty. By integrating both model-based robust design approach and data-based uncertainty estimation, the proposed design approaches are able to design the nonlinear system to be robust not only to deterministic/probabilistic variation of parameters but also to model uncertainty.
4. Three novel robust eigenvalue design approaches are proposed to minimize the influence of parameter variations to eigenvalues of the dynamic system. First, several modeling methods, first-order modeling, multiregion modeling, and SN modeling, can effectively approximate the eigenvalue variation respectively for linear system, approximately linear system, or nonlinear system under parameter variations. Then, the developed sensitivity design method could effectively handle the complex sensitivity matrix problem derived from the obtained model. The solution obtained can guarantee stability and robustness of the designed system under parameter variations.
5. A novel robust design is proposed to achieve stability and robustness of the system when model uncertainty exists. This proposed robust design is to minimize the influence of parameter variations and model uncertainty to system eigenvalues. By integrating the stability theory and the perturbation theory, the method can effectively design the system to be stable and robust under model

uncertainty. Moreover, the tolerance design can further achieve the best performance under given constraints as long as design variables stay in the tolerance space.

6. A new design-for-control-based integration method is presented for the nonlinear process with hybrid discrete/continuous variable. The approach can effectively obtain a well linear approximation for the complex system, so that the controller design can be simplified under the robust pole placement. The advantage is that the designed system can be controlled effectively by a relatively simple and easy-realization controller. More importantly, this integrated design and control approach can easily achieve the satisfactory robust pole placement since it has combined the merits of both robust design and control.
7. A novel intelligence-based integration method is proposed for the hybrid system—a nonlinear process with unmeasured overall performance working in a large operating region. The fuzzy modeling method can approximate the nonlinear process very well in a large operating region and the PSO-based optimization method can effectively handle the nonconvex and nondifferential integration problem. This proposed integration design can achieve the desired overall performance of the production as well as local dynamics of the nonlinear system.

Effectiveness of the presented design approaches is verified on selected cases in manufacturing industry. These methods and approaches could be applicable to a wide range of industrial applications.

11.2 CHALLENGE

In addition to the progress achieved so far, there are still many challenges in robust design and control integration, which are discussed in the following examples.

1. ***Robust optimal design under model uncertainty.*** Though the proposed robust designs can achieve the satisfactory performance, they do not consider performance optimization under model uncertainty. The solutions achieved are robust but may not be optimal. It would be a challenge to incorporate the robust design into a proper optimization approach so that solutions are not only robust but also optimal.
2. ***Robust design for the strongly nonlinear system under model uncertainty.*** Since performance variations are usually approximated by the linear model in the existing robust designs, although the higher order term neglected may be regarded as model uncertainty, the proposed method will be conservative when the system is strongly nonlinear. Thus, an effective robust design should be developed for the strongly nonlinear system under model uncertainty.
3. ***Integration of design and control for the partially controllable system.*** For the partially controllable system, only controllable variables can be directly

adjusted online and uncontrollable variables have to rely on process design. In difference to the fully controllable system, the design of this system has two different tasks. One is to have the robust performance of uncontrollable variables through the off-line design, and the other is to adjust controllable variables online to have the optimal control performance. Since these control and design variables are coupled in the complex process, such an integration for the robust and optimal performance will be a long-standing challenge in future research.

4. ***Robust design for the nonlinear distributed parameter system (DPS) with model uncertainty and parameter variation.*** The nonlinear DPS usually has strong spatio-temporal, or infinite-dimensional dynamics, which can be described by nonlinear parabolic partial differential equations (PDEs) with complex boundary conditions. The input, output, and even parameters of DPS can vary both temporally and spatially. All the existing design methods have not been applied to DPS for parameter design due to the overcomplexity of this kind of system. It would be a great challenge and an extremely difficult endeavor to achieve a feasible robust design for DPS.
5. ***Integration of design and control for the DPS with model uncertainty and parameter variation.*** Design variables and controllable variables can significantly change process dynamics. The design variables are discrete variables, while the controllable variables are continuous variables. However, the discrete design and continuous control have not been properly integrated to optimize the performance even for the lumped process, let alone DPS. It is even a bigger challenge to design and control these factors to have an optimal performance for DPS.

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